Impedance matching networks

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THERE are many times when experimenting with various electrical circuits at communication frequencies that it is necessary to connect two circuits together in such a way that each of them works into or out of some correct terminating impedance. These terminating impedances may have almost any values, but by far the most usual condition is when the termination has an impedance equal to the impedance of the circuit or network itself. When the impedances of two circuits are the same, such as two similar telephone lines, it is only necessary to connect them directly together, and the condition is fulfilled. Often, however, the proper terminating impedance for one circuit is not correct for the other. In this case, some special kind of coupling device is required.

As an example, the output of a vacuum-tube is to be coupled to a telephone line in the most efficient possible manner. The impedances of the two circuits in this case may be widely different. The output circuit of the amplifier may have an impedance of, for instance, 2,000 ohms. The line impedance may be only 500 ohms. If the output from the plate circuit were connected directly to the line, the resulting impedance mis-match would cause a reduction in the power delivered to the load, (termed "reflection loss"[1]), of 1.58 to one. In logarithmic units, it is two decibels. This loss may be calculated by a formula which gives the reflection loss directly in decibels, in terms of the two impedances.

$$N_{DB} = 20 \log_{10} \frac{R_G + R_L}{\sqrt{4 R_G R_L}}$$

Where N_{DB} = the attenuation in decibels, R_G = the generator impedance, and R_L = the load impedance, or in this special case, the plate impedance of the tube and the line impedance respectively.

To reduce this loss, it is common practice to make use of a device called an impedance-matching transformer. This is a transformer having a turns ratio between the primary and secondary windings equal to the square root of the ratio of the impedances that it is to couple.

Aside from the power loss that results from the mis-matching of impedances, there are many other cases when correct operating conditions are obtained only when proper terminating impedances are used. For example, if a calibrated attenuation network is not properly terminated its calibration is worthless, unless a correction term is applied. Loud-speakers, audio frequency amplifying transformers, and such instruments are all designed for correct operation from a circuit of some definite impedance. For all of these uses, the right impedance-matching transformer may not always be available. To build one up is a lengthy and tedious job. Fortunately a most convenient substitute is available. This is a simple resistive network called an impedance-tapering network or "taper pad." It can easily be built up of two decade resistance boxes. Once the resistance values have been decided upon for any particular circuit, fixed resistors may be substituted.

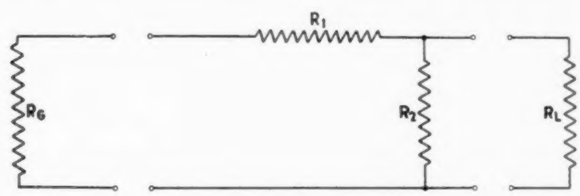

Fig. 1—Taper pad circuit diagram

A taper pad has a number of definite advantages. It has a fixed and known loss, it is not affected by frequency to the extent that a transformer may be, and the values can be readily and accurately determined.

IT does not always happen that an engineer finds himself equipped with the proper terminating impedance for the circuit upon which he is working. How to calculate the proper network to terminate this circuit, and how to build it up are discussed, briefly, in this article.

Calculating a taper pad

Figure 1 shows a taper pad at the junction of a generator whose impedance is of some value, R_G, and a load, R_L. It is desired to calculate the value of the series branch, R_1, and the shunt branch, R_2, so that both the generator and the load see their respective impedances, R_G and R_L, looking toward the junction. Assume R_G to be the greater to fix the direction that the network points.

Looking from the generator, R_G, there is a combination of resistances to be equal to R_G, so

$$R_G = R_1 + \frac{R_2 R_L}{R_2 + R_L}$$

Looking from the load, R_L, there are the resistances to be equal to R_L; thus

$$R_L = \frac{R_2(R_1 + R_G)}{R_1 + R_2 + R_G}$$

Here are two simultaneous equations with two unknowns, R_1 and R_2. Solving them (which is a straightforward if laborious process) the following result is obtained:

$$R_2 = \frac{R_L R_G}{\sqrt{R_G(R_G - R_L)}} \qquad (1)$$

Knowing the generator and load impedances, this is immediately solved. Then

$$R_1 = R_G - \frac{R_2 R_L}{R_2 + R_L}$$

and by a similar process:

$$R_1 = \sqrt{R_G\ (R_G - R_L)} \qquad (2)$$

There are, thus, two general equations which permit the calculation of an impedance-tapering network to match any two impedances. They are rigorous, and assuming that the constants of the generator and load are definitely known the result obtained is exact.

Losses introduced by network

Such a network always introduces some loss in the circuit. It is a function of the ratio of the two impedances and may be calculated from the formula,

$$1/2\left(\frac{n+1}{n}\right) = \sqrt{\frac{R_G}{R_L}}$$

Let $\sqrt{R_G/R_L} = \rho$, and the expression becomes

$$n^2 - 2\rho n + 1 = 0$$
$$\text{and} \quad n = \rho + \sqrt{\rho^2 - 1} \qquad (3)$$

The loss, n, is the ratio of the voltage or current that would appear in the load, R_L, if the load were coupled to the generator, R_G, by an ideal impedance-matching transformer, to the voltage or current that actually appears in the load when the resistive taper pad is in the circuit. An ideal impedance-matching transformer is the most efficient conceivable coupling device. It neither stores nor dissipates energy.[2] Its transformation ratio is such that when its secondary is connected to the load, its primary impedance is exactly equal to the generator impedance. The ratio of the power that can be delivered to the load through such an ideal transformer, to the power actually delivered, is the accepted way to measure the efficiency of an impedance-matching device. We have taken this ratio in terms of voltage or current and called it n. The arithmetic ratio, n, may be expressed logarithmically in decibels:

$$N_{DB} = 20 \log_{10} n$$

The fact that this loss is so definitely known is often a great help when calculating the total gain or loss of a circuit. (See Fig. 2.)

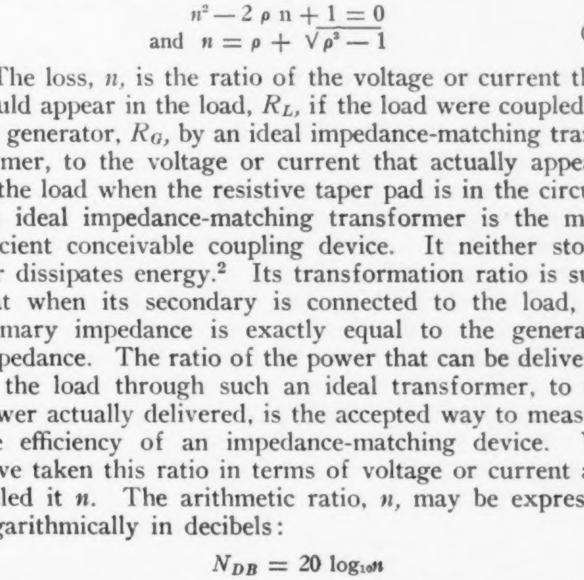

Fig. 2—Loss in network as function of impedances to be matched

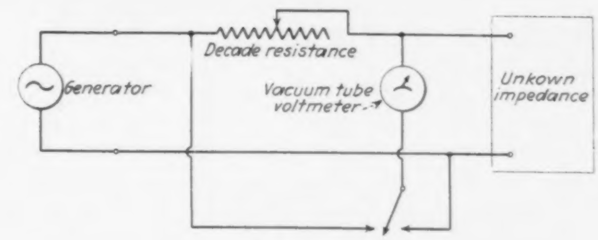

Fig. 3—Method of measuring impedance by means of decade resistance box and vacuum tube voltmeter

There is one question that arises in this connection, and that is, if the impedances to be matched are not definitely known, how can they be determined.

To measure an unknown impedance

There are, of course, a great variety of bridge measurements that can be made, and these are probably the most accurate methods. The proper bridges are not always at hand, however, and the method may be unduly complicated. Figure 3 shows one very convenient method of finding the impedance of a circuit. It is quite accurate at the audio frequencies but stray admittances, unsuspected mutual couplings, and other errors are apt to creep in at radio frequencies which makes the method unsuitable if high accuracy is necessary. The resistance, R, is an ordinary decade-resistance box. The vacuum-tube voltmeter, the input impedance of which must be very high with respect to the circuit being measured, is used to compare voltage across the unknown to that across the decade box. The box is adjusted until the voltage is the same across both circuits. With this condition, since the current is the same through both the known and unknown units, the impedance of the unknown is equal to the resistance setting of the decade box. It must be remembered that the circuit impedance so measured includes both the resistive and the reactive component. If it is necessary to separate these components which, as was previously mentioned, is not usually important, a calibrated inductance or capacitance is required. If the unknown is inductive the capacitance, of course, is used, and vice versa. It is placed in series with the unknown, the voltmeter placed across both, and the reactance adjusted until the voltmeter reads a minimum. When this is done the reactive component has been canceled and all that remains is the resistance. The decade resistance is then adjusted as before until the voltage reading is the same across the two circuits. The resistance of the calibrated reactance must be subtracted from the result to give the net resistance of the unknown circuit.

It is hoped that these remarks about the design and use of resistive taper networks will bring to the attention of experimenters a very useful, and somewhat neglected tool, for working with those circuits in which impedance matching is a considerable factor. In problems connected with the transmission, recording, and reproducing of the voice frequencies the question is often of the greatest importance.

[1] "Transmission Networks and Wave Filters," by T. E. Shea (D. Van Nostrand Company, Inc., 1929) contains a most complete discussion of this question. It must be noted that in order to discuss the problem in the most general way, the phase angle of both the generator and the load must be considered. Fortunately, considering the impedance as purely resistive is a close enough approximation for most practical cases.

[2] K. S. Johnson, "Transmission Circuits for Telephone Communication," Chap. IV, has a complete discussion of ideal transformers.