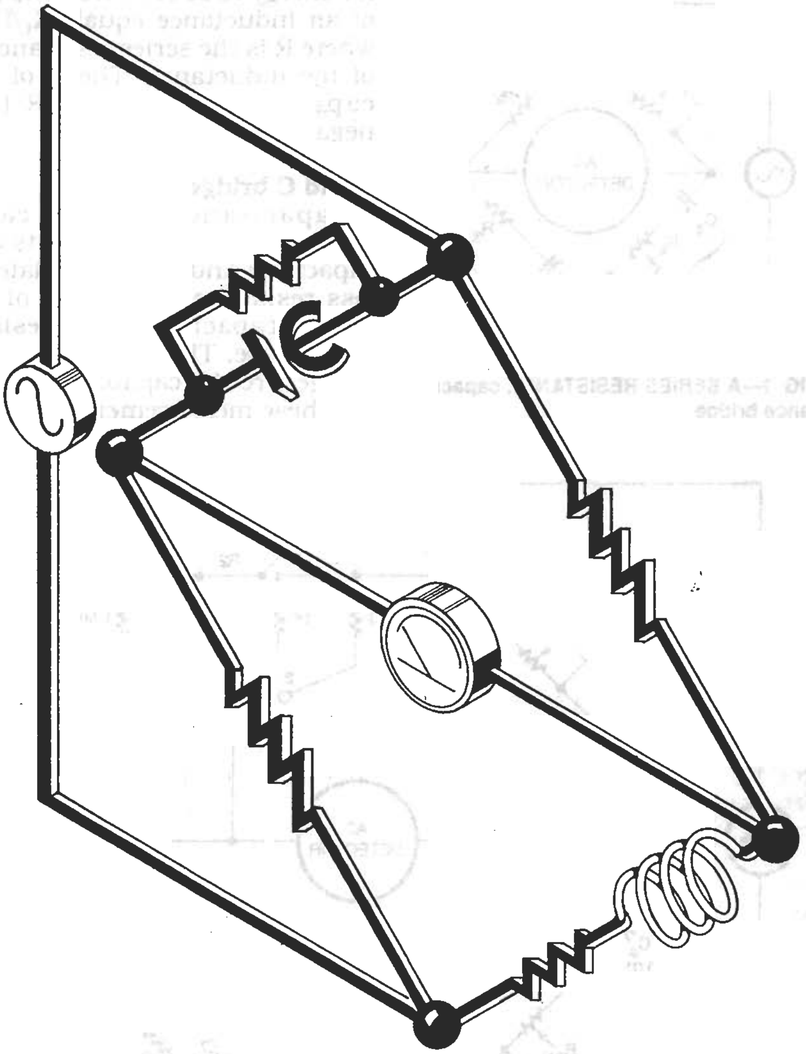


L-C BRIDGES

Learn about popular capacitance and inductance bridges, and build your own Hay-Maxwell bridge and an 18-range LCR laboratory-grade bridge.

RAY MARSTON

THIS ARTICLE IS ABOUT AC BRIDGES for measuring inductance and capacitance. It explains one popular capacitance bridge and two popular inductance bridges. Included are schematics for building your own AC bridges and modifying them for your requirements in experiments or making precise laboratory-grade measurements. The previous article in this series on bridges explained the basic Wheatstone DC resistance bridge and modifications to that bridge that made it possible to measure both inductance and capacitance.

Reactance, impedance, and Q

Inductors and capacitors offer resistance to alternating current. In a capacitor, resistance is known as capacitive reactance, and in an inductor it is known as inductive reactance. Reactance depends on frequency, and the capacitor's value farads and the inductor's value in henrys. The equation for capacitive reactance is:

$X_C = 1/(2\pi fC)$

The equation for inductive reactance is:

$X_L = 2\pi fL$

The term $2\pi f$ is called *angular frequency* and is represented by the lower-case Greek letter omega (ω).

It is important to remember that neither practical capacitors nor inductors are pure reactances because of the presence of mechanical elements that introduce parasitic inductance and resistance into capacitors and parasitic capacitance and resistance into inductors.

However, the true equivalent circuits for inductors and capacitors can be simplified for accurate measurements of most components used in general applications, but all of these residual effects must be taken into account in measuring some components such as high-value aluminum electrolytic capacitors.

Because of phase relationships in AC circuits, current is seldom in phase with voltage. In a pure capacitor, current leads voltage by a phase angle of 90°, and in a pure inductive circuit voltage leads current by a phase angle of 90°. Phase angle is designated by the lower case Greek leter theta (θ). These angle relationships change with the introduction of other reactive components.

Impedance is the result of resistance and reactance in an AC circuit. The phase angle θ between the voltage and current can be any angle between −90° and +90°.

Impedance in a series circuit is:

$$Z = \sqrt{R^2+X^2}$$
$$\theta = \tan^{-1} X/R$$

Impedance in a parallel circuit is:

$$Z = RX/\sqrt{R^2+X^2}$$
$$\theta = \tan^{-1} X/R$$

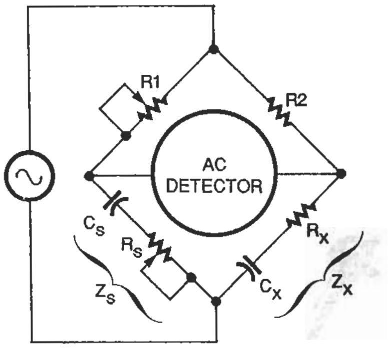


FIG. 1—A SERIES RESISTANCE capacitance bridge

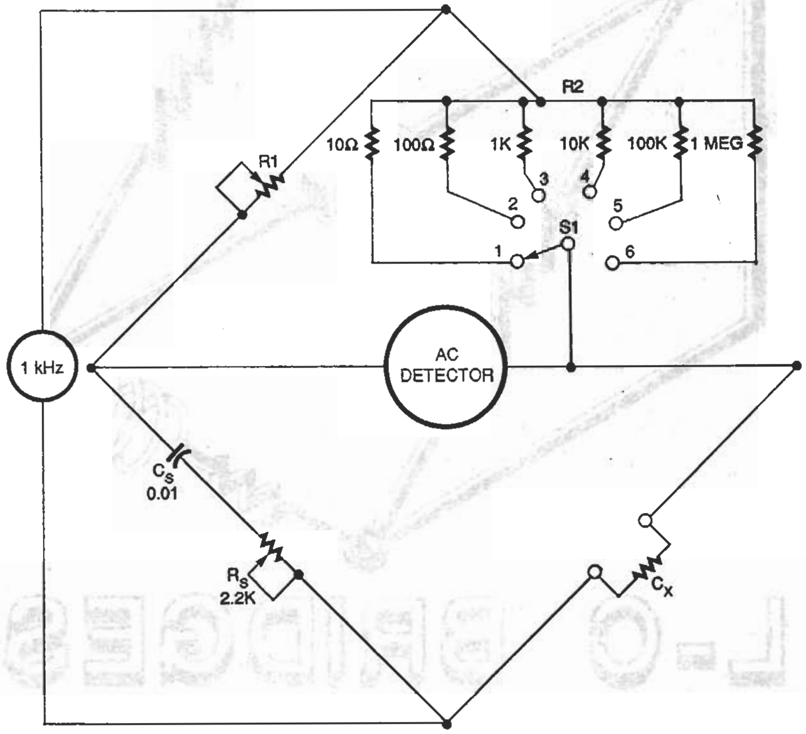


FIG. 2—SIX RANGE SERIES resistance-capacitance bridge.

TABLE 1
RESISTANCE VS. CAPACITANCE RANGE VALUES
RESISTANCE-CAPACITANCE BRIDGE

Switch S1 Range	Bridge Range	Resistor R2 Value	Z_X Full Scale R_S=0
1	0 - 10 μF	10 Ω	**15.9 Ω**
2	0 - 1.0 μF	100 Ω	**159 Ω**
3	0 - 0.1 μF	1.0 K	**1.59 K**
4	0 - 0.01 μF	10 K	**15.9 K**
5	0 - 1.0 μF	100 K	**159 K**
6	0 - 100 pF	1 MEG	**1.59 MEG**

The Q of a capacitor, coil or device is a figure of merit for its energy-storing capability. The higher the Q, the longer it takes for energy to be released. The Q of an inductance equals X_L/R, where R is the series resistance of the inductance. The Q of a capacitor equals $-X_C/R$ (a negative value).

L and C bridges

Capacitance bridges can make precise measurements of capacitors and their associated loss resistances in terms of a known capacitance and resistance value. There are different bridge circuits capable of making these measurements.

There are also many different bridge circuits that can measure inductance because the impedance of each arm can be a combination of resistances, inductances, and capacitances. One popular capacitance bridge and two popular inductance bridges are discussed later in this article.

Series RC bridge

The schematic for the series RC bridge is shown as Fig. 1. It is a resistance-ratio bridge that compares a known capacitance with an unknown capacitance. The bridge is balanced when $R1/Z_S = R1/Z_X$. Under this condition:

$C_X = C_S(R1/R2)$
$R_X = R_S(R2/R1)$
$Q = 1/2\pi f C_X \times R_S$

The values of R1 and R_S balance the AC voltages and phase shifts on the detector's left legs with those on its right legs.

Figure 2 is the schematic for a six-range series RC bridge that spans the range of 1 picofarad to 10 microfarads in six decade ranges. About half of the source 1kHz voltage appears at each end of the detector at balance. The AC detector can be either a simple meter or headphones. Table 1 relates the six capacitance ranges switched by S1 with the resistance value in each channel and full-scale impedance value.

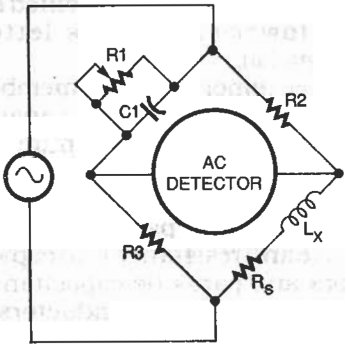


FIG. 3—THE MAXWELL BRIDGE can measure the values of low-Q coils.

Variable resistor R_S permits a null to be obtained when capacitor C_X has Q values as high as 7.2. (Any capacitor with a Q value that high should be discarded.) The value of potentiometer R_S can be calibrated directly in 1/Q values because in this circuit with a 1-kHz source, 1/Q = 0.001 per 15.9 ohms of

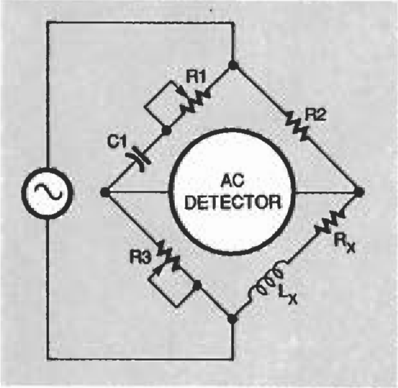
Continued on page 87

BRIDGE CIRCUITS
continued from page 44

the resistance value of the bridge's potentiometer R_S.

Inductance bridges

The two most popular bridges for measuring inductance are the *Maxwell bridge* (also known as the *Maxwell-Wien* bridge), shown in Fig 3 and the *Hay* bridge (also known as the *opposite-angle* bridge), shown in Fig. 4. Both operate on the principle of balancing the inductive phase shift of the unknown inductor L_X against a capacitive shift of the same magnitude in the opposite arm of the bridge.



FIG, 4—THE HAY BRIDGE can measure the values of high-Q coils.

TABLE 3
RESISTANCE VS. R, C, and L RANGE VALUES
LABORATORY STANDARD BRIDGE

Switch S2 Range	Resistor R2 Value	Bridge Range Resistance	Capacitance	Inductance
1	10 Ω	0 - 10 Ω	0 - 10 µF	0 - 1 mH
2	00 Ω	0 - 159 Ω	0 - 1 µF	0 - 10 mH
3	1.0 K	0 - 1.59K	0 - 0.1 µF	0 - 100 mH
4	10 K	0 - 15.9 K	0 - 0.01 µF	0 - 1 H
5	100 K	0 - 159 K	0 - 1.0 µF	0 - 10 H
6	1 MEG	0 - 1.59 MEG	0 - 100 pF	0 - 100 H

The Maxwell bridge, shown in Fig. 3, is widely used for accurate inductance measurements. It determines unknown inductance values with a standard capacitor, which has an advantage over a standard inductor because it is less likely to be influenced by external fields and is easier to shield. Moreover, the field set up by a capacitor is negligible. Standard capacitors are small and inexpensive. The Maxwell bridge is useful for measuring coils with Q values below 10. The important Maxwell bridge formula's are:

$L_X = R2 \times R3 \times C1$
$R_X = (R2 \times R3)/R1$
$Q_X = 2\pi f(L_X/R_X)$
$= 2\pi fC1 \times R1$

Note: All Q values are influenced by the source frequency. Thus, a coil that has a Q of 100 with a high source frequency might have a Q of 1 or less at 1 kHz.

The *Hay bridge* is similar to the Maxwell bridge and is used for measuring inductances that have large values of Q (greater than 10) or whose resistance is a small fraction of the reactance X_L. The Hay bridge can also determine the incremental inductance of iron-cored reactors. The important formulas for the Hay bridge are:

$Q_X = 1/2\pi f \times C1 \times R1$
$L_X = C1 \times R1 \times R3 \times (1/1 + 1/Q^2$
$R_X = 2\pi f \times L_X/Q$

Figure 5 is the schematic for a combined Hay and Maxwell inductance bridge that spans the range of 10 microhenrys to 100 henrys in six ranges. The Hay circuit makes high-Q measurements when switch S1-a and S1-b re set in the "H" position. When switch S1-a and S1-b is set in the "L" or Maxwell position, the bridge measures low values of Q.

In this circuit, AC voltages at both ends of the detector approach the half-supply value at balance. The AC detector here can also be a simple device such as headphones. Table 2 relates the selected switch channel to the inductance range, related

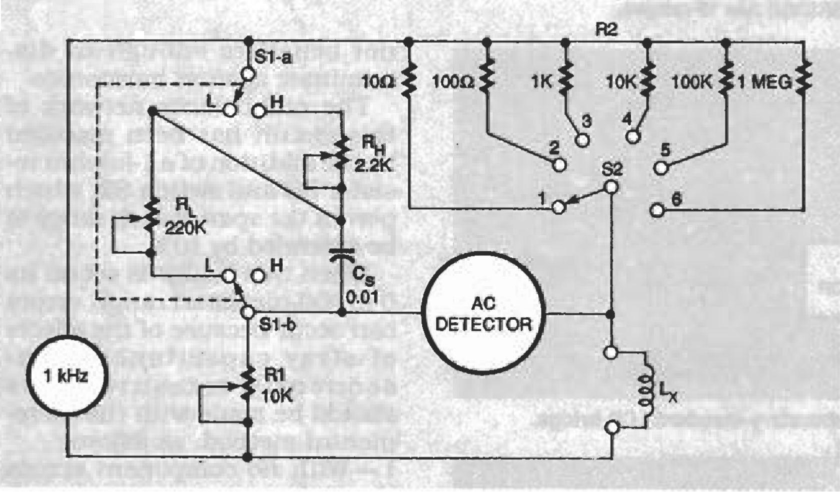


FIG. 5—THIS HAY-MAXWELL BRIDGE has six ranges.

TABLE 2
RESISTANCE VS. INDUCTANCE RANGE VALUES
HAY/MAXWELL INDUCTANCE BRIDGE

Switch S1 Range	Bridge Range	Resistor R2 Value	Z_X Full Scale 1 kHz
1	0 - 1 mH	10 Ω	15.9 Ω
2	0 - 10 mH	100 Ω	159 Ω
3	0 - 100 mH	1.0 K	1.59 K
4	0 - 1 H	10 K	15.9 K
5	0 - 10 H	100 K	159 K
6	0 - 100 H	1 MEG	1.59 MEG

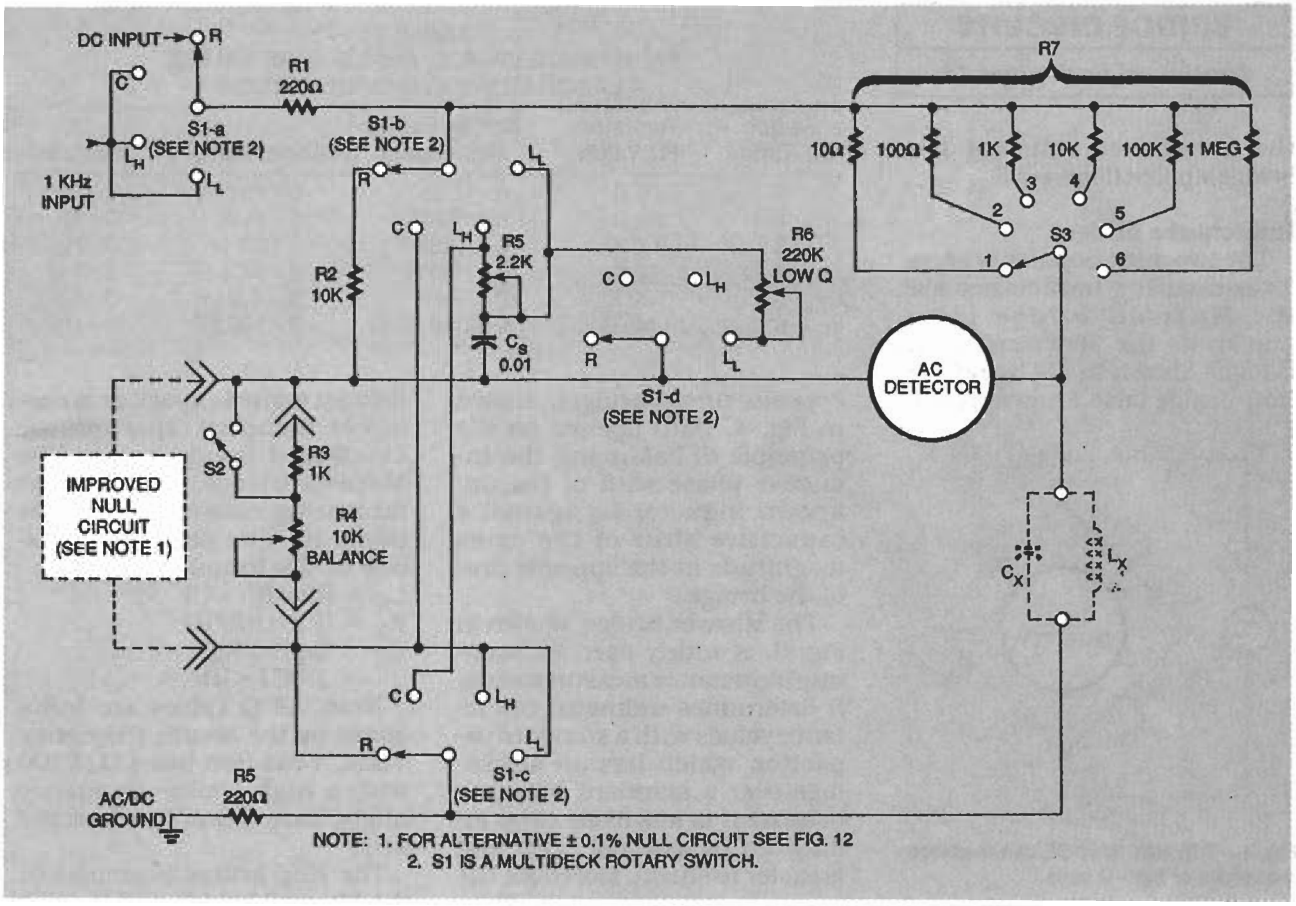


FIG. 6—THIS LABORATORY-STANDARD LCR BRIDGE has 18-ranges.

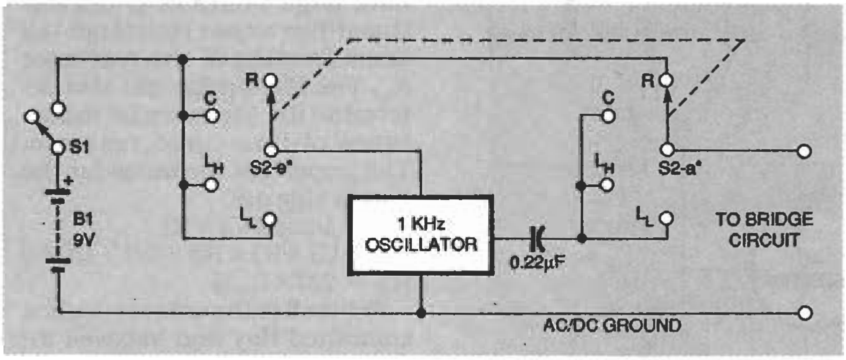


FIG. 7—AN AC/DC SOURCE for the 18-range laboratory-standard LCR bridge.

resistance value, and full-scale impedance.

Precision LCR bridge

Figure 6 is the schematic for a precision 18-range LCR bridge that combines the circuits of Figs. 5 and 2 with a Wheatstone bridge. Although this bridge is very sensitive, it requires only a simple AC balance detector. This could be an analog voltammeter on its DC-sourced resistance ranges, or a headphone on its the AC-sourced capacitance and inductance ranges. A simple radio receiver will also work when the ear is not sensitive enough to discriminate against harmonics.

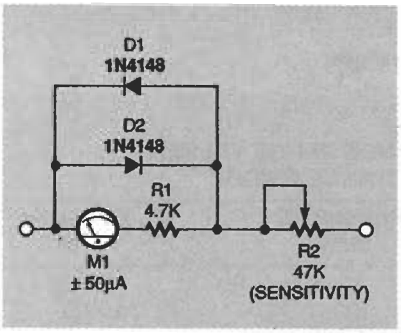


FIG. 8—A DC-NULL DETECTION circuit.

The null-balance network of this circuit has been modified by the addition of a 1-kilohm resistor R3 and switch S2, which permit the span of each range to be extended by 10%.

When this bridge is set on its 0 to 100-picofarad range, errors can occur because of the effects of stray capacitance. Consequently, measurements should be made with the *incremental* method, as follows:

1.—With no component across the unknown terminals, null the bridge with potentiometer R4 and record the resulting *residual* null reading (typically about 15 pF).

2—Insert the unknown capacitor in place, obtain a balance reading (e.g., 83 pF), and then subtract the residual value (e,g., 15 pF) to obtain the true capacitor value (in this example, 68 pF).

Table 3 relates the switch range to the resistor value in the channel selected and the vari-

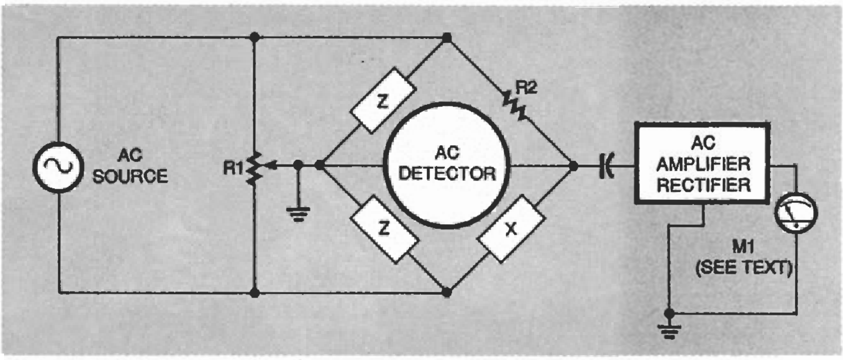


FIG. 9—A WAGNER EARTH CONNECTION in a single-ended AC null detector.

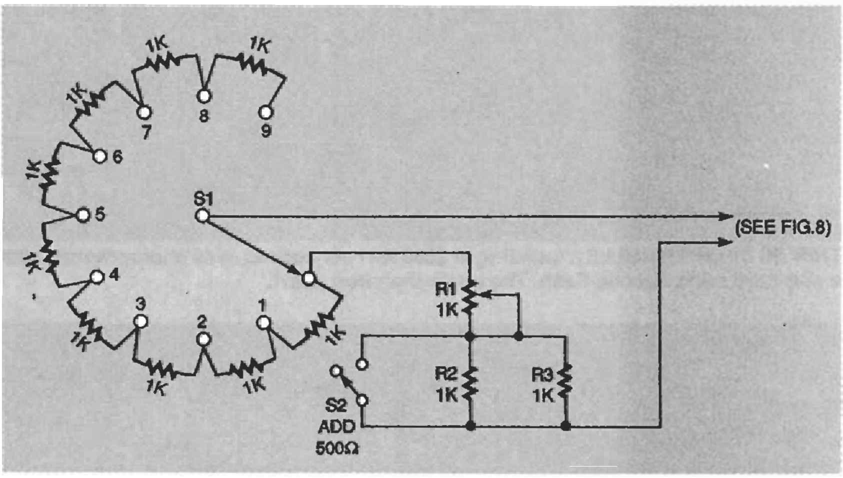


FIG. 10—CIRCUIT FOR IMPROVING null resolution of the LCR bridge to ± 0.1% of full scale.

ous ranges of resistance, capacitance, and inductance.

The bridge circuit in Fig. 6 can be built as shown for use with external AC and DC sources and null-detection circuitry, or can be modified to suit your specific requirements. Fig. 7 shows an AC/DC source that can be added if a five-wafer multideck rotary switch is installed in Fig. 6 and the "e" wafer is left blank.

Both DC and AC null-balance detectors can easily be included in the Fig. 6 circuit. The DC detector can be a microammeter capable of measuring ±50 microamperes. Figure 8 is the schematic for a microammeter protection circuit consisting of two back-to-back 1N4148 silicon diodes and R2, a 47-kilohm potentiometer for adjusting sensitivity.

Figure 9 shows a simplified schematic for an AC detector. It includes a single-ended AC analog millivoltmeter. The low input of the detector and the detector junction on the left side of the bridge can both be grounded. This diagram also shows how the AC source can be equipped with a *Wagner earth connection*, supplied by potentiometer R1. This potentiometer permits the source signal to be balanced to ground to eliminate unwanted signal interference at null.

Figure 10 is the schematic for a null resolution circuit that can be installed in the LCR bridge, shown in Fig. 6. The bridge has a resolution that is only about ±1% of full scale. This is set by the limit of readability of the R3 and R4 balance control's scale.

The circuit in Fig. 10 will improve the bridge's resolution by a factor of 10 to ±0.1% of full-scale. Remove Switch S2, resistor R3 and potentiometer R4 and install the switched and variable network of Fig. 10 at the male and female connectors shown on Fig. 6. Switch S2 permits the 1-kilohm linear potentiometer R1 to go overrange by 50%. Ω