Testing logic networks

New method using numerical spectra can be implemented on a computer

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As digital equipment becomes more complicated there is an increasing need for effective fault diagnosis techniques. This article introduces the subject of Rademacher-Walsh spectra, which can be used in the design of logic networks but have not so far been applied to network testing. The speed with which Rademacher-Walsh spectra may be handled by a digital computer may be of great significance.

While "fault detection" is a go/no-go test to check whether a network performs its required input-output functions correctly, "fault diagnosis" is a more comprehensive test to (ideally) pin-point the source of the fault if one exists. Automatic fault diagnosis is the aim of modern digital testing techniques.

For automatic testing it is normally assumed that faults in logic circuits will be logical faults, that is logic 0 signals are erroneously present instead of logic 1 signals, or vice versa, and that such faults are steady and not intermittent. This implies that a faulty logic network has an input-output behaviour which is logical but not the correct one, and in general a fault may be defined by "stuck-at-1" or "stuck-at-0" to indicate a faulty logic node in the network.

One objective of automatic fault diagnosis is to examine the network under test with the minimum number of test sequences. This involves applying a chosen series of input conditions such that all paths in the network from input to output are verified. Fundamental difficulties may be experienced in practice which no input-output testing can resolve, such as internal faults which do not propagate to the output (which may arise for example in redundant or hazard-free network designs), and faults on different nodes which give rise to the same output fault. However, accepting such fundamental constraints, the test diagnosis methods currently in use are based upon conventional Boolean techniques; such methods are largely derived from the theoretical development of Seshu (1956), Roth (1966), Kautz (1968) and Sellers *et. al.* (1968)[1, 2, 3, 4]. These are well reviewed in recent summary papers[5, 6, 7].

However, instead of truth-tables or Boolean equations to express the input-output relationships of any given logic network, there is a completely different method by which such input-output relationships may be expressed. This is the Rademacher-Walsh spectral method. In the following sections this method is briefly introduced and an indication given of further research in this area.

The Rademacher-Walsh spectra

The Rademacher-Walsh spectrum of any given logic function with n independent input variables, say x_1 to x_n, consists of a series of 2^n integer numbers. The magnitude and sign of these 2^n numbers constitute the spectral coefficient values of the given function. They uniquely define the given function[8], as would a Boolean truth-table, but in a completelv dissimilar manner from the 2^n entries of 0 and 1 in a truth-table.

The rigorous mathematical background and possible use of these spectral coefficients in logic synthesis will be found documented[9, 10, 11]. However, here it will suffice to state what the spectral coefficient values represent, and how the values may be calculated by hand for simple functions. The actual coefficient values may differ in sign or by some normalising factor between different authorities, depending upon initial definitions, but the following gives a straightforward definition based upon logic 0 and logic 1 values without any normalising.

Consider a logic network with n independent binary inputs x_1 to x_n. Then the 2^n Rademacher-Walsh spectral coefficients which characterise such a network are labelled: $R_0, R_1, R_2, \ldots, R_n, R_{12}, R_{13}, \ldots, R_{12\ldots n}$, where the subscripts are all possible combinations of subscripts 1 to n taken one-at-a-time, two-at-a-time, etc. up to n-at-a-time, in addition to the first subscript R_0. For example, for $n=4$ we have the sixteen coefficients: $R_0, R_1, R_2, R_3, R_4, R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}, R_{123}, R_{124}, R_{134}, R_{234}, R_{1234}$

The spectral coefficient values for any given Boolean function $f(x_1, \ldots, x_n)$ may be digitally-computed extremely rapidly by an appropriate fast Walsh transform which has been developed[12]. However, hand-computation, which helps to emphasise the meaning and numerical significance of each coefficient value, may be undertaken as follows.

For illustration, take the simple three-variable function $f(x) = [x_1\bar{x}_2 + x_2\bar{x}_3]$. Compile the truth-table shown in Table 1, which as well as listing the given inputs x_1, x_2, x_3, also lists $x_0, x_1 \oplus x_2, x_1 \oplus x_3, x_2 \oplus x_3$, and $x_1 \oplus x_2 \oplus x_3$, where $x_0 \equiv$ always 0, $x_1 \oplus x_2$ is the exclusive – OR of inputs x_1, x_2, and so on. This "primary set" and "secondary set" constitute an augmented set of input variables. They are conventionally labelled r_0, r_1, r_2 etc., as shown in Table 1. The function output is listed in the normal manner.

Table 1. The truth-table for function $f(x) = [x_1\bar{x}_2 + x_2\bar{x}_3]$ in terms of the augmented set of input variables r_i. (Note: the total number of variables r_i will always be 2^n for any n).

"Primary" input set				"Secondary" input set				Function output
$r_0 = x_0$	$r_1 = x_1$	$r_2 = x_2$	$r_3 = x_3$	$r_{12} = x_1 \oplus x_2$	$r_{13} = x_1 \oplus x_3$	$r_{23} = x_2 \oplus x_3$	$r_{123} = x_1 \oplus x_2 \oplus x_3$	$f(x)$
0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	1	1	0
0	0	1	0	1	0	1	1	1
0	0	1	1	1	1	0	0	0
0	1	0	0	1	1	0	1	1
0	1	0	1	1	0	1	0	1
0	1	1	0	0	1	1	0	1
0	1	1	1	0	0	0	1	0

Now the Rademacher-Walsh spectral coefficients for any given function $f(x)$ are given by:

$R_i, i=0,1,2,\ldots, 12\ldots n$, = (Number of agreements between the value 0 or 1 of the input r_i and the output $f(x)$) − (number of disagreements between r_i and $f(x)$).

For the function tabulated in Table 1, this gives:

R_0	R_1	R_2	R_3	R_{12}	R_{13}	R_{23}	R_{123}
0	+4	0	−4	+4	0	+4	0

These spectral coefficient values are illustrated in Fig. 1. This figure is the spectrum of the given function, and may be considered analogous to the familiar Fourier frequency spectrum of a complex analogue waveform.

Points of note are:

- the given function is uniquely defined by the value of the spectral coefficients[8]. Hence if any fault develops in the network which propagates to the output, then the spectral coefficient values for the network will be modified;
- the relative magnitude of each coefficient is a measure of the "importance" of the particular input parameter $r_i, i\neq 0$, in determining the network output value 0 to 1. A high positive value indicates a high degree of dependence of the output on the particular r_i input, while a high negative value indicates a high degree of dependence of the output on the complement of the particular r_i input, i.e. upon $\bar{r}_i$.

For example, in the function illustrated in Fig. 1 the output of the network is relatively highly dependent upon r_1 (=input x_1), not very dependent upon r_1(=input x_2), and relatively highly dependent upon $\bar{r}_3$(=input $\bar{x}_3$). The further spectral coefficients give a measure of the importance of r_{12}, r_{13} etc. (=the exclusive—ORs of the inputs x_1,x_2, etc) in controlling the output value.

Further, for fault diagnosis, suppose a "healthy" network has a certain high-value spectral coefficient, and under some internal fault condition a low value is found for this coefficient, then such a discrepancy indicates that the particular node(s) or path(s) of the network which contributes to this part of the spectrum is suspect.

These ideas, therefore, are the basic ideas underlying the possible use of Rademacher-Walsh spectra for logic-network fault diagnosis.

Basic properties of the spectra

Certain basic properties of the spectra of a logic network may be detailed:

R_0 maximum-valued. If the value for R_0 for a network is found to be $\pm 2^n$ (that is ± 8 for a 3-variable system, ± 16 for a 4-variable system and so on), then the network output is constant, that is stuck-at-0 or stuck-at-1. For a 4-variable system, a stuck-at-0 output gives $R_0=+16$, and a stuck-at-1 output gives $R_0=-16$, using the definition of R_0 given in the preceding section.

With $R_0=\pm 2^n$, all other spectral coefficient values will be zero. There is therefore no need to evaluate them once this maximum value condition for R_0 is known.

R_j maximum-valued. If the value for any primary coefficient R_j, $j=1$ to n (that is the spectral coefficient value relating directly to a binary input $x_1,x_2,\ldots, x_n$), is found to be $\pm 2^n$, then the output of the network is controlled entirely by the one particular input x_j. All other inputs are redundant or ineffective in controlling the network output. For example should R_2 be +16 in a logic network with four input variables x_1 to x_4, then the network output is entirely dependent upon x_2, the output being $f(x)=x_2$. If R_2 was −16 then output $f(x)=\bar{x}_2$.

With any primary coefficient R_j at a maximum value, then all other spectral coefficients, including R_0, will be zero. There is no need to evaluate them once a maximum condition for R_j is known.

R_{jk} maximum-valued. If the value for any secondary coefficient $R_{jk}\ldots$, $j\neq k$, $j,k=1,2\ldots$, (that is the spectral coefficient values relating to the exclusive—ORs of the binary inputs $x_1,x_2,\ldots, x_n$), is found to be $\pm 2^n$, then the output from the network is controlled entirely by the exclusive—OR of two (or more) inputs x_j,x_k. For example, should R_{234} be + 16 in a network with four input variables x_1 to x_4, then the network output is entirely dependent upon $x_2\oplus x_3\oplus x_4$, x_1 being redundant or ineffective. If R_{234} was −16, the output would be $[\overline{x_2\oplus x_3\oplus x_4}]$.

With any secondary coefficient $R_{jk}\ldots$ at a maximum value, then all other spectral coefficients, including R_0, will be zero.

R_0 zero-valued. No particular significance may be attached to the situation where R_0 is zero. This only indicates that the output from the network is 1 for exactly the same total number of input minterms as when the output of 0. For example, see the simple network evaluated above. R_0=zero does not give any indication which input minterms give the network output 0 or 1.

R_j zero-valued. No particular significance may be attached to any one R_j, $j=1$ to n, being zero-valued. This only indicates a certain symmetry in the network output 0 and 1 values with respect to the 0 and 1 values of the particular x_j input. For example, see R_2 in the example given above.

R_{jk} zero-valued. Similarly no particular significance may be attached to any one secondary coefficient value being zero-valued. However, see the following case.

Multiple zero-valued coefficients. While no great significance may be attached to a single zero-valued coefficient, more than one zero-valued coefficient may have significance.

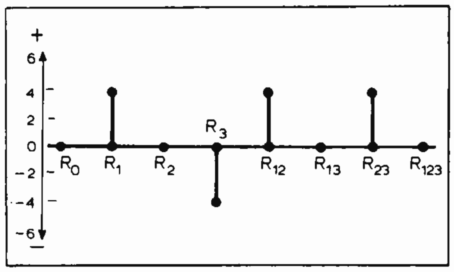


Fig. 1. The Rademacher-Walsh spectrum of the Boolean function $f(x)=[x_1\bar{x}_2+x_2\bar{x}_3]$

If the value of any primary coefficient $R_j, j=1$ to n, is zero, and if all the secondary coefficients R_{jk}, R_{jl}, etc. containing this coefficient subscript j are also zero, then (and only then) the network input x_j is redundant or ineffective.

For example, if the spectrum for the function shown in Fig. 1 had been evaluated assuming four inputs x_1,x_2,x_3 and x_4 were present, x_4 in this particular example being unnecessary (redundant), then (i) the value of all the spectral coefficients shown in Fig. 1 would each be twice as great, corresponding to the twice-as-many agreements/disagreements now present in the truth-table, but (ii) the coefficients for $R_4, R_{14}, R_{24}, R_{34}, R_{124}, R_{134}, ,R_{234}$ and R_{1234} would all be zero-valued.

It will further be noticed that if a necessary input to a logic network becomes stuck-at-0 or stuck-at-1 to the complete network, this fault will result in all the associated spectral coefficients becoming zero-valued.

Certain other restricted zero-valued relationships may be formulated. However these restricted zero-valued relationships do not have such a fundamental importance as cases where all coefficients containing a particular subscript identification are zero.

Relationships of the spectral coefficient values. Collectively the spectral coefficient values define the input-output relationships of the complete network. The zero-value and the maximum-value coefficients have a particular significance, as briefly covered above.

However, in every case there are certain arithmetic relationships which exist between the values of all the coefficients, but these relationships are indirect.* Further, it is known that the coefficient values for any function must lie in a very restricted set of numbers, the actual ordering and signs being the specific spectrum for the given function. For example, there are only eight possible sets of spectral coefficient

*By the very definition of these coefficient values, one is not free to alter the value of any one without automatically altering the value of others. So the freedom in the range these numbers can collectively take is restricted, but in a difficult way to non-mathematically express and appreciate.

magnitudes (the "positive canonic spectra") to cover all possible logic networks with four input variables[13].

The full significance, understanding, and potential usefulness of these considerations are the subject of continuing research. Statistical considerations can come into this area, as it is not necessary to evaluate more than a certain percentage of the total spectrum in order to define certain functions with a high degree of surety. As far as faults within networks are concerned, then it would appear that any fault which propagates to the output will cause a dramatic change in the resulting spectrum, and not a minor change in, say, one of the higher-ordered spectral coefficients only. Thus it may not be necessary to consider all the spectrum in fault-diagnosis procedures.

Effect of logic inversion and logic gates on spectra

It would be attractive to be able to furnish a simple set of arithmetic rules detailing how the spectral coefficient values build up as one considers the progression through a logic network. Unfortunately no such set of rules is available to cover all situations. The following, however, details some simple features for particular cases.

Logic inversion. A logic inversion (NOT) of a Boolean function changes the sign of all spectral coefficient values. Thus the spectrum of a function before and after an inverter gate is related by sign changes only, which mathematically may be regarded as multiplying the spectrum by −1.0. Similarly, the output spectrum from a NAND gate is −1.0 times that of an AND gate with the same inputs, and that of a NOR gate is −1.0 times that of an OR gate.

Fig. 2. Disjoint inputs to a 3-input OR gate.
Input 1 $=\bar{x}_1\bar{x}_2\bar{x}_3$
Input 2 $=\bar{x}_1 x_2 x_3$
Input 3 $= x_1 x_3 \bar{x}_4$
Output z $= [\bar{x}_2\bar{x}_2\bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 x_3 \bar{x}_4]$

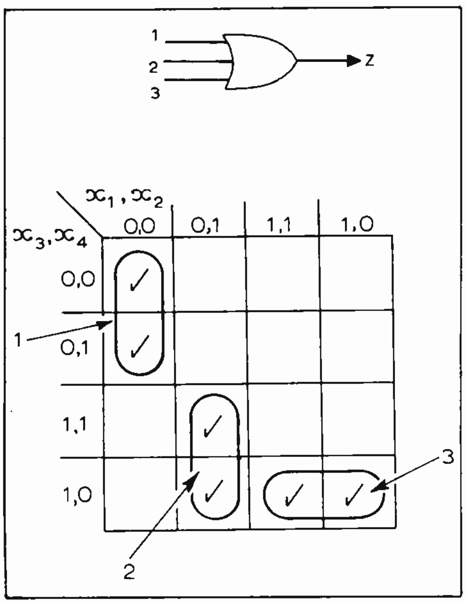


Table 2.

	R_0	R_1	R_2	R_3	R_4	R_{12}	R_{13}	R_{14}	R_{23}	R_{24}	R_{34}	R_{123}	R_{124}	R_{134}	R_{234}	R_{1234}
Input 1:	+12	−4	−4	−4	0	−4	−4	0	−4	0	0	−4	0	0	0	0
Input 2:	+12	−4	+4	+4	0	+4	+4	0	−4	0	0	−4	0	0	0	0
Input 3:	+12	+4	0	+4	−4	0	−4	+4	0	0	+4	0	0	−4	0	0

and the output spectrum is:

+4	−4	0	+4	−4	0	−4	+4	−8	0	+4	−8	0	−4	0	0

Table 3.

	R_0	R_1	R_2	R_3	R_4	R_{12}	R_{13}	R_{14}	R_{23}	R_{24}	R_{34}	R_{123}	R_{124}	R_{134}	R_{234}	R_{1234}
Input 1:	−12	+4	+4	+4	0	+4	+4	0	+4	0	0	+4	0	0	0	0
Input 2:	−12	+4	−4	−4	0	−4	−4	0	+4	0	0	+4	0	0	0	0
Input 3:	−12	−4	0	−4	+4	0	+4	−4	0	0	−4	0	0	+4	0	0
Output:	−4	+4	0	−4	+4	0	+4	−4	+8	0	−4	+8	0	+4	0	0

This example is illustrated in Fig. 4.

Output spectrum of an OR gate. Neglecting for the moment the R_0 term, which as we shall see is a special case, then if and only if the input signals to an OR gate are disjoint (that is they have no logic 1 minterms in common) the output spectrum of the gate is given by the arithmetic sum of the individual input spectra.

The R_0 spectral components do not obey this simple arithmetic summation; instead the R_0 spectral component of the gate output is given by

$$\left\{ \sum_{1}^{m} (\text{input } R_0\text{s}) + (m-1)2^n, \right\}$$

where m is the number of inputs to the OR gate. The reason for this exception can be demonstrated very simply.

As an example, a three-input ($m=3$) OR gate with the inputs shown in Fig. 2 is in the category of an OR gate with disjoint inputs. The spectra are given in Table 2.

However, when the input signals are not disjoint, and "overlap" occurs between logic 1 input minterms, then simple addition of the input spectral component values is not valid. This, unfortunately, is more often the case than not in logic networks, unless the designer deliberately sets out to make all inputs disjoint.

Output spectrum of a NOR gate. The situation with a NOR gate is precisely that of the OR gate, except for the output inversion which is equivalent to multiplication of the output spectrum by −1.0. Again, therefore, only if the input signals are disjoint can simple addition of the input spectra be made.

Output spectrum of an AND gate. The AND relationship can be generated by inverter and NOR gates as shown in Fig. 3. Therefore if, and only if, the complements of the inputs to an AND gate are disjoint, the output spectrum of the gate excepting R_0 may be obtained by the arithmetic sum of the individual input spectra. The output R_0 value is given by

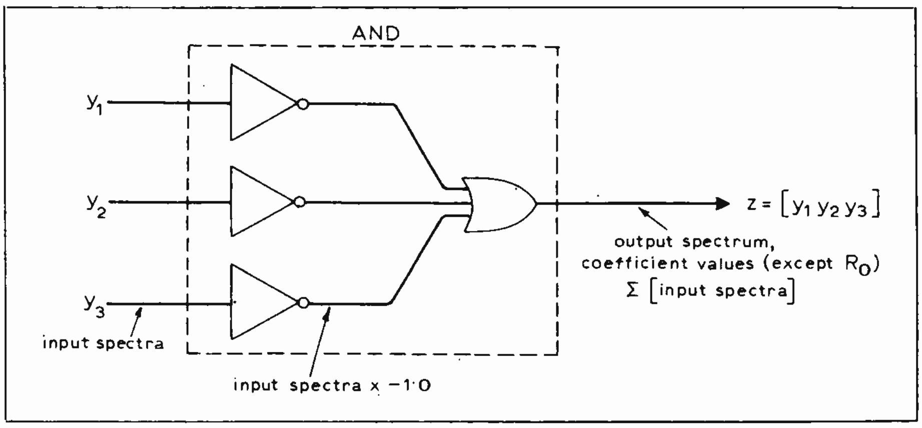
$$\left\{ \sum_{1}^{m} (\text{input } R_0\text{s}) + (m-1)2^n \right\}$$

As an example, taking the complements of the inputs used in the previous disjoint OR of Fig. 2 as inputs to a 3-input AND gate, see Table 3.

Output spectrum of a NAND gate. The situation with a NAND gate is precisely that of the AND gate, except for the multiplication by −1.0 of the output spectrum.

Thus all these OR and NOR, and AND and NAND cases now considered are special cases, and are therefore of restricted use. For the more general case, the numerical relationships between spectra before and after a logic gate are given by matrix manipulations, which can readily be handled by a digital computer. The cases here considered with their simple arithmetic relationships are particular cases of such manipulations.

Fig. 3. The equivalent of an AND gate.



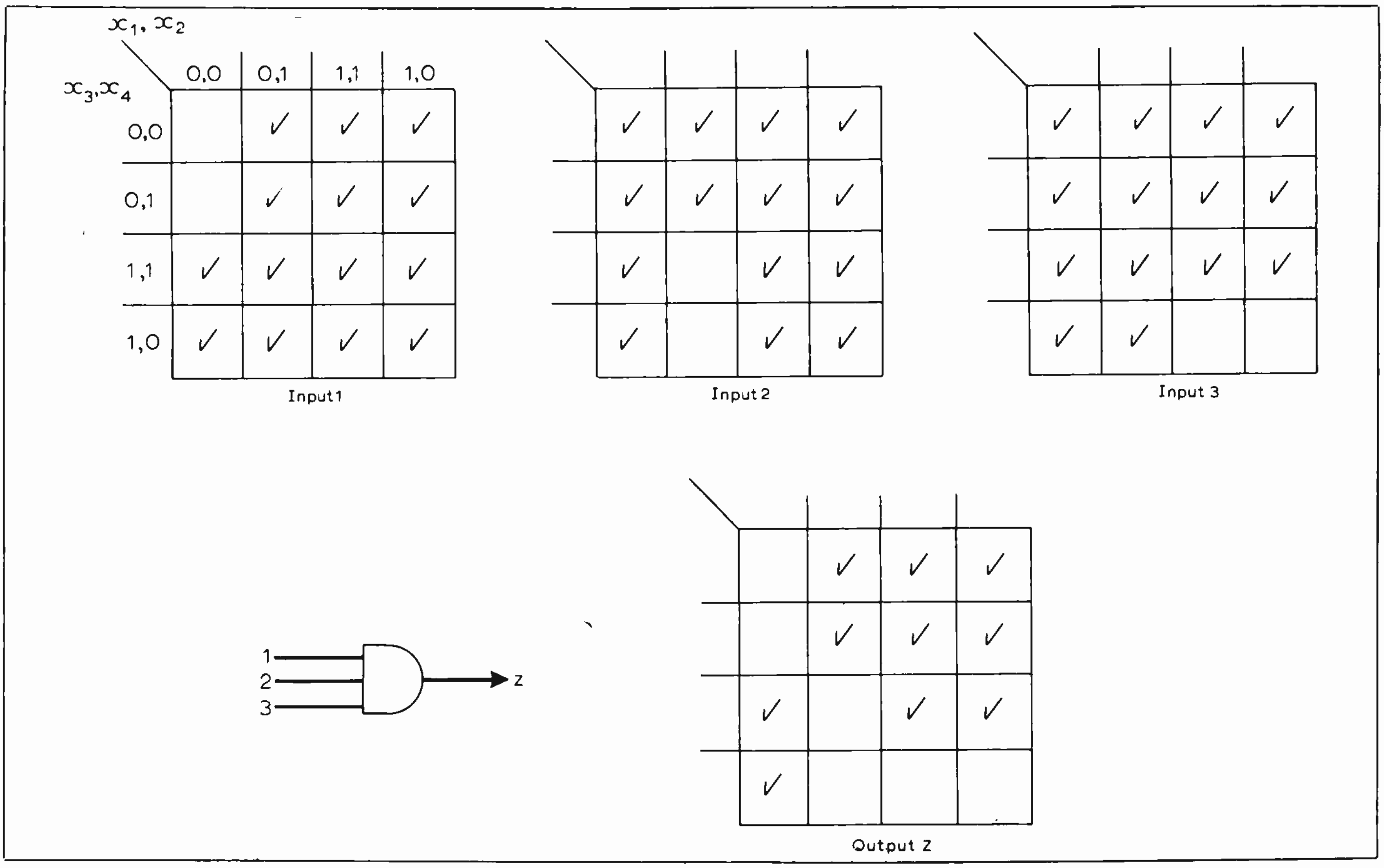


Fig. 4. Inputs to a 3-input AND gate, with disjoint complementary inputs.
Input 1 $= x_1+x_2+x_3 = \mathrm{NOT}\bar{x}_1\bar{x}_2\bar{x}_3$
Input 2 $= x_1+\bar{x}_2+\bar{x}_3 = \mathrm{NOT}\bar{x}_2x_2x_3$
Input 3 $= \bar{x}_1+\bar{x}_3+x_4 = \mathrm{NOT}x_1x_3\bar{x}_4$
Output $z = [(x_1+x_2+x_3)\,(x_1+\bar{x}_2+x_3)\,(\bar{x}_1+\bar{x}_3+x_4)]$

The effect of logical faults, an example

Any logical fault in a network which propagates to the output will cause a change in the output spectrum. However the change is unlikely to be a simple arithmetic change, except in restricted cases such as considered above.

As an exercise, consider the simple all-NAND circuit shown in Fig. 5. This circuit deliberately has no strong characteristics or disjoint properties which might give rise to simple manipulations of spectral coefficient values. It is a network not strongly dependent upon any one or more input parameter.

The fault-free spectral coefficient values at nodes (c), (k) and (o) and at the output are as shown in Table 4 (note, + signs have now been omitted for brevity):

A computer-run of the output spectrum with each individual node stuck-at-0 and stuck-at-1 yields the output coefficient values under each single-fault condition as shown in Table 5.

From this tabulation it will be noted that all faults which are logically distinguishable at the output give rise to a distinct output spectrum. In this particular network it is sufficient to examine the primary spectral components R_0 to R_4 only to diagnose the fault classification, the information contained in the remaining components being unnecessary. However further research is necessary to show whether this is always so.

Summary and further research

The underlying mathematics of the Rademacher-Walsh transform, and the advantages of computer-handling of data in the spectral domain as compared with the Boolean domain, have not been covered, as these may be found in existing literature. The software for computing the spectral component values for given circuits, both with and without "stuck-at" faults present, is straightforward and fast.

At this early stage of research, the outcome and potential of this technique in fault diagnosis lies in the future.

Fig. 5. NAND network, output $z = [x_1x_2+\bar{x}_1\bar{x}_2\bar{x}_3+x_2x_3x_4]$ *with internal nodes (a) to (o) as the source of potential stuck-at-1 or stuck-at-0 faults.*

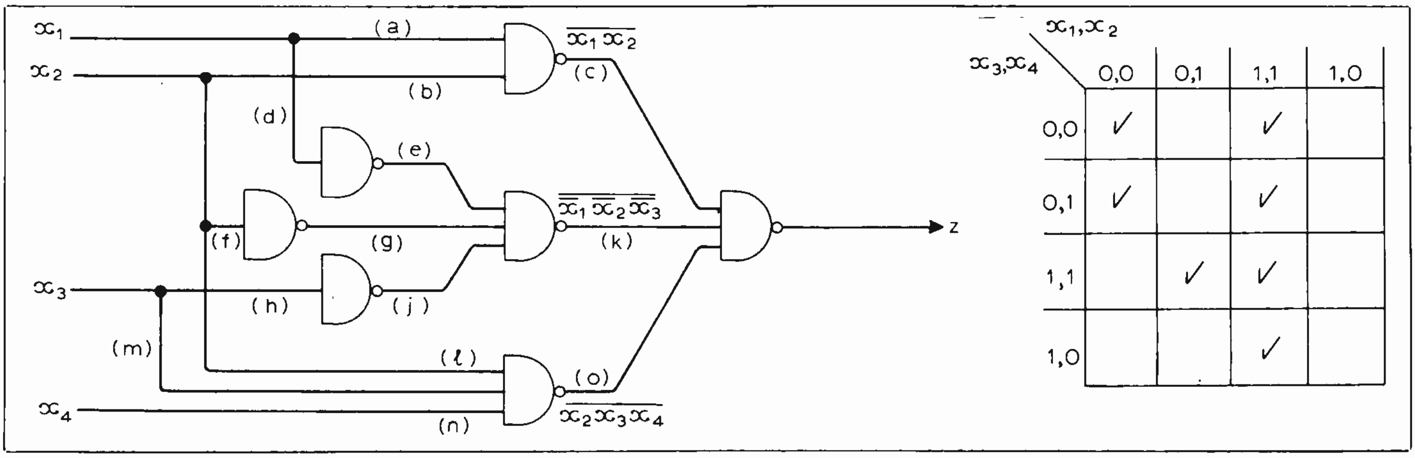


Table 4.

	R_0	R_1	R_2	R_3	R_4	R_{12}	R_{13}	R_{14}	R_{23}	R_{24}	R_{34}	R_{123}	R_{124}	R_{134}	R_{234}	R_{1234}
Node (c):	−8	−8	−8	0	0	8	0	0	0	0	0	0	0	0	0	0
Node (k):	−12	4	4	4	0	4	4	0	4	0	0	4	0	0	0	0
Node (o):	−12	0	−4	−4	−4	0	0	0	4	4	4	0	0	0	−4	0
Output z:	2	2	6	−2	2	−10	−2	2	−6	−2	−2	−6	−2	−2	2	2

Table 5.

(a) or (b) stuck-at-0, or (c) stuck-at-1	8	−4	0	0	4	−4	−4	0	−8	−4	−4	−4	0	0	4	0
(d), (f) or (h) stuck-at-1, or (e), (g) or (j) stuck-at-0, or (k) stuck-at-1	6	6	10	2	2	−6	2	2	−2	−2	−2	−2	−2	−2	2	2
(l), (m) or (n) stuck-at-0, or (o) stuck-at-1	4	4	4	−4	0	−12	−4	0	−4	0	0	−4	0	0	0	0
(a) stuck-at-1	−4	−4	12	−4	0	−4	−4	0	−4	0	0	−4	0	0	0	0
(b) stuck-at-1	−6	10	−2	−2	2	−2	−2	2	−6	−2	−2	−6	−2	−2	2	2
(d) stuck-at-0, or (e) stuck-at-1	−2	6	2	−6	2	−6	2	2	−10	−2	−2	−2	−2	−2	2	2
(f) stuck-at-0, or (g) stuck-at-1	−2	−2	10	−6	2	−6	−6	2	−2	−2	−2	−2	−2	−2	2	2
(h) stuck-at-0, or (j) stuck-at-1	−2	−2	2	2	2	−14	2	2	−2	−2	−2	−2	−2	−2	2	2
(l) stuck-at-1	−2	2	2	2	6	−10	−2	2	−2	2	−6	−6	−2	−2	−2	2
(m) stuck-at-1	0	0	8	−4	4	−8	−4	4	−4	−4	0	−4	−4	0	0	0
(n) stuck-at-1	0	0	8	0	0	−8	0	0	−8	0	0	−8	0	0	0	0
(c), (k) or (o) stuck-at-0	−16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fundamentally the spectra contain a very high and inter-related information content, which must be usable in many ways apart from network synthesis which has been its main research use to date. For fault diagnosis, the following are some of the immediate problems under consideration:

- Does the spectrum of a fault-free network itself provide sufficient or partial data for determining diagnostic test sequences?
- Can the spectrum of a faulty network be used to control automatic diagnostic procedures by controlling the input test sequences?
- Because all logic functions may be grouped into positive canonic spectra classfications, can standard test techniques be outlined for each such group of functions?
- Is it possible to compute a useful "partial spectrum" by considering only a limited number of "agreements — disagreements" (see above), instead of all 2^n?
- How best can the network-under-test/computer-interface be engineered?
- Can useful all-hardware (not computer-based) field test sets be formulated, for testing relatively small digital networks[14]?
- Do these ideas, even if satisfactory, give any advantages over existing digital a.t.e. techniques?

It is hoped that this introductory article may stimulate continuing research in this new area.

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turns off and has no further effect on the VU circuitry. Diodes D_3 and D_4 isolate the two VU circuits from each other and also prevent Tr_{10} from being driven into conduction on negative half-cycles.

A further point about the VU system, which should have been emphasized in the original article, is that the 10kΩ resistor and germanium diode associated with the meter itself are only relevant if inexpensive milliammeter movements are fitted. If professional VU meters are used, which contain internal bridge rectifiers, the above components are unnecessary.

Because the amplifier is already fitted with a relay that switches off the main outputs, it is simple to add a socket for remote muting. Fig.4 shows part of the original rumble-gate system, together with the new components required. All that is involved is the closure of a switch between the base and emitter of Tr_{29} so that the transistor turns off and causes the relay contacts to open. Note that, like the switch-on delay, this facility overrides all other control functions. The resistor and diode are included to protect the circuit if a wrong connection to the remote-muting socket is made. As the control lead only handles a small direct voltage the audio signal cannot be degraded.

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