Secrets of Transmission Lines

Part 3: More AC review.

John A. Kuecken KE2QJ
2 Round Trail Dr.
Pittsford NY 14534

In part 2, we discussed the effects of the inductor and the capacitor, with the inductor storing energy in the magnetic field and the capacitor storing energy in the electrostatic field. By itself, neither of these effects dissipates any energy. When the magnetic field of the inductor collapses and when the capacitor discharges, all of the stored energy is given back (at least theoretically, in perfect devices). If the pendulum used as an example were operated in a vacuum so that there would be no air resistance to the swinging, and if the mount and suspension did not flex, the pendulum would swing on forever. Note that this is not "perpetual motion," in that no energy or work is extracted. It is simply a system in which no (or at least very little) energy is being dissipated, just as the Earth will continue to orbit the Sun, if not forever, at least for a very long time.

If you used a plumb bob weighing 62 pounds suspended by a 220-foot length of steel music wire with a swing arc of 10 feet, you would find that the pendulum would swing for several days from the initial impulse. This was the arrangement used by J.-B.-L. Foucault to demonstrate the rotation of the Earth. The plane in which the pendulum swings would slowly rotate in azimuth. At the north pole it would make a complete rotation in a day, and at lower latitudes it would rotate more slowly, falling to zero at the equator.

The point is that there is no real power dissipated in the imaginary components of an impedance. This point deserves a little more explanation, and is perhaps best visualized by the graph in **Fig. 1.** From our part 1 discussion of Ohm's law, we saw that power is the product of voltage times current. For the alternating current, from equation (2-8):

V = Vo*[sin(wt)]
eqn (3-1)

and for an inductor, from (2-11):

i = [-Vo/(w*L)]*cos(w*t)
(3-2)

Thus:

Power in inductor = -Vo*{[sin(w*t)] /(w*L)} watts
(3-3)

The plot of this equation is shown in **Fig. 1** with the crosshatched area. To simplify, we assumed (w*L) = 1. You can see that, averaged over a half cycle, the power is zero since the negative part cancels the positive part. What the inductor absorbs in the first half, it gives back in the second half.

Not so the case for a resistor. From Ohm's law, we can obtain the current through a resistor as:

i = Vo*[sin(w*t)/R] amperes
(3-4)

where
R = resistance in ohms

Multiplying by the voltage to get power, we obtain:

Power = [Vo*sin(w*t)]*{Vo*[sin(w*t)] /R} watts
(3-5)

Thus:

Power = $(Vo^2)*[\sin^2(w*t)/R]$

This curve is also plotted in the lower half of **Fig. 1**. Note that because the sine function is squared, it never

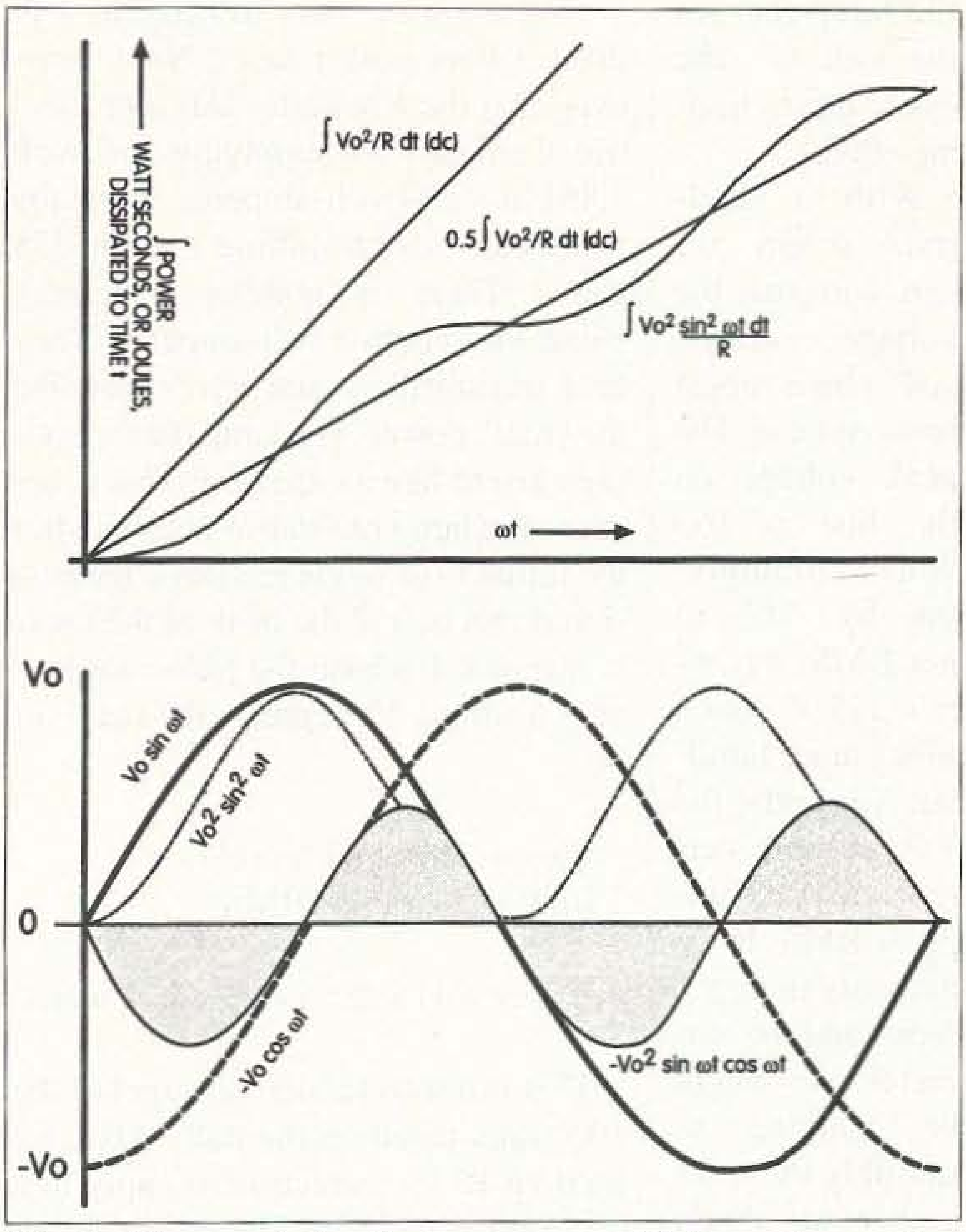

Fig. 1. The cumulative heating power with time of a resistor driven by an alternating voltage.

goes negative. This is real power that makes the resistor hot.

As shown in the $\sin^2$ curve, the instantaneous power in the AC case occurs in two peaks per cycle, one when the voltage is maximum positive and the other when it is maximum negative. In the upper half of the figure, we show how the joules or watt-seconds accumulate for two DC cases and the AC case. If the DC voltage is equal to Vo, the power accumulates faster than in the AC case; however, at 0.5 times the DC rate, the accumulation is equal on the average. If the energy were applied to a resistor or an oven, the heating would be equal. This value of voltage is termed the Root Mean Squared value, usually written RMS or rms.

Vrms = [sqr(0.5)]*Vo = 0.707*Vo (3-6)

Note that this numeric relationship between the peak AC voltage, Vo, and the RMS voltage applies only to sine waves; other waveforms have other relationships.

A similar relationship can be used to show that a similar effect applies to a capacitor. The current flowing in the capacitor represents no real power.

Power factor and phase angle

All real inductors and capacitors have some loss associated with them. Therefore, the lossless circuit, where

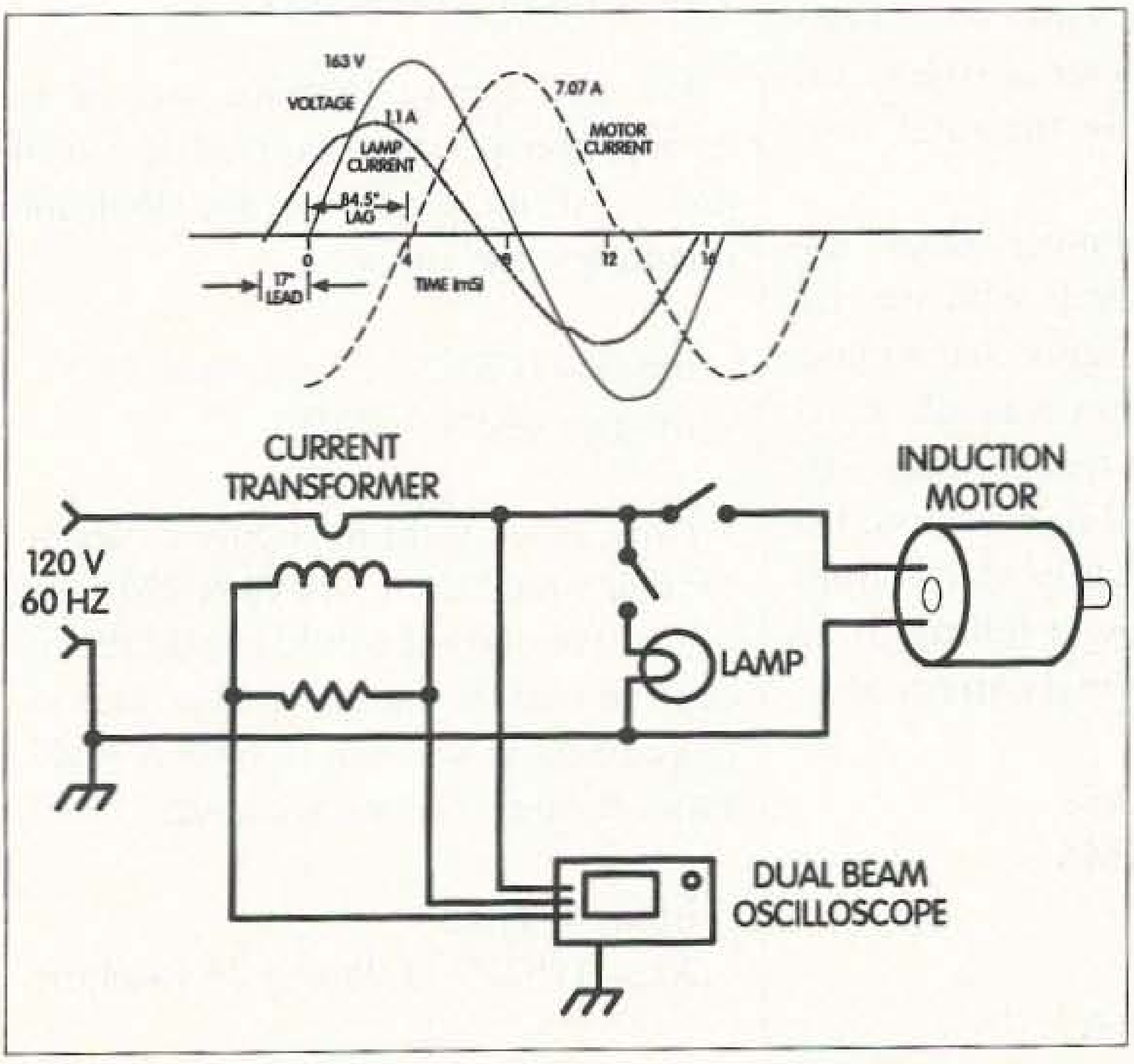

Fig. 2. Current and voltage monitoring of an induction motor.

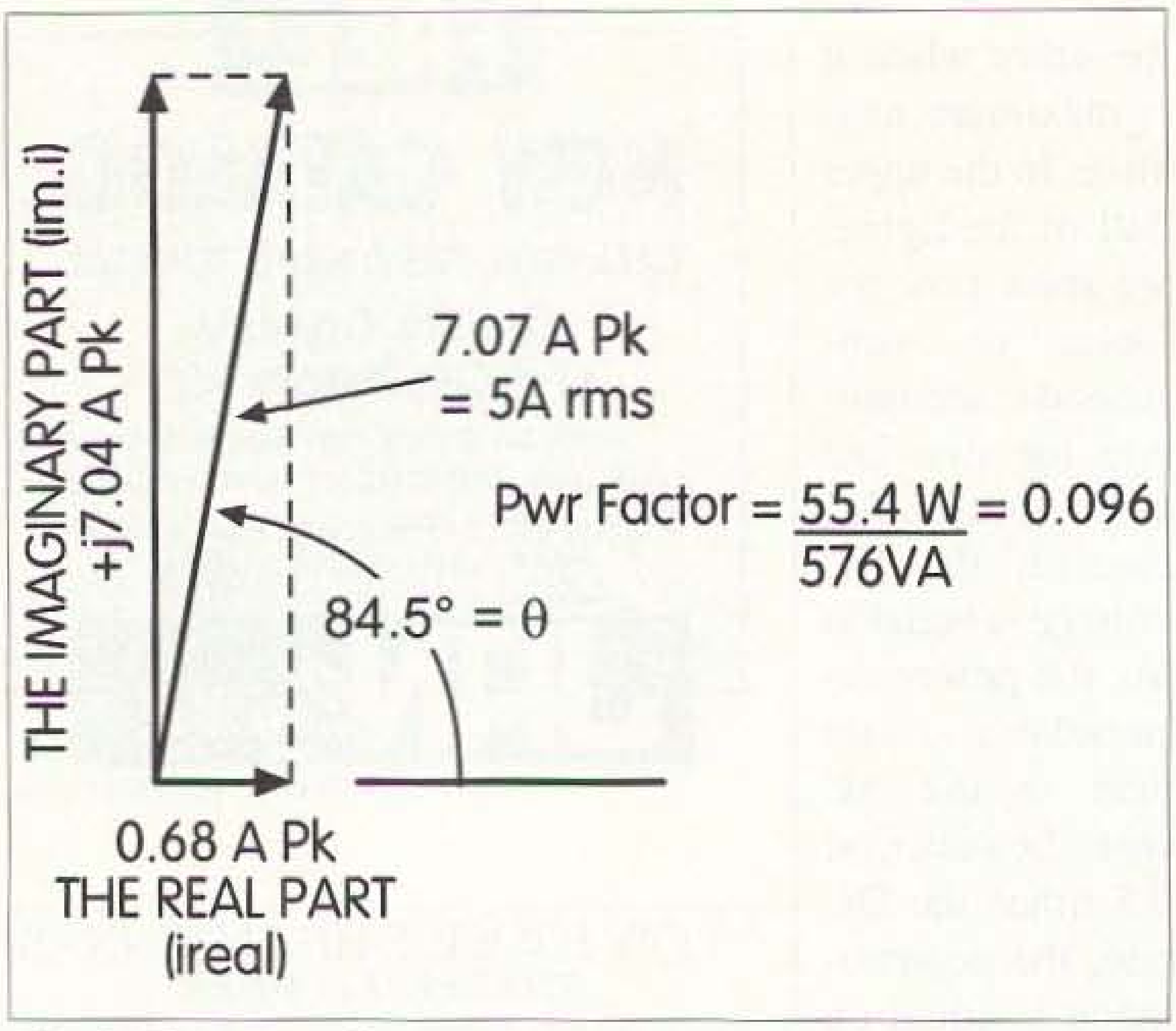

Fig. 3. The vector diagram.
Eqn (3-7) ipeak = irms/0.7071
(3-8) 0.7071(7.07) = 5 Arms
(3-9) ix (real) = ix cos θ
ix (imaginary) = jix sin θ
(3-10) Power (real) = irms (real) Vrms watts
Power (imaginary) = irms (imaginary) Vrms VARs
(3-11) Power (real) = 0.7071 (0.68) x 0.7071 (163) = 55.4 watts or 0.074 horsepower
Power (imaginary) = 0.5 (j 7.04) 163 = 574 VARs

the inductor and capacitor simply exchange energy without loss, does not exist. Let us examine a simple practical case. The curves of **Fig. 2** represent measurements made on equipment in my shop. The motor and lamp are both on a drill press.

The motor is rated at 1/3 horsepower and the lamp is rated at 100 watts. For the data in the illustration, the motor is more or less loafing, just turning itself and the tapered roller bearing quill in the drill press. In all likelihood, the main power loss is in turning the belt.

We will describe how to make the current transformer later. Suffice it to say here that the transformer can be calibrated to read so many volts per ampere and the phase angle is zero. That is, the output voltage is precisely in phase with the current (not the voltage) on the line under measurement.

Both current waveforms are slightly distorted from perfect sine waves. In the case of the induction motor, the distortion is at the crossover point and probably due to hysteresis effects. In the case of the lamp, the distortion is due to the fact that the resistance of the lamp changes throughout the cycle due to heating effects.

With a dual-trace scope, we can compare the voltage, current, and phase angle between them. The peak voltage on the line is 163 volts. Multiplying by .707 to get RMS, we obtain 115 V, which looks more familiar. Similarly, the 7.07 A peak current yields 5 amperes RMS. If we had only the voltmeter and the ammeter, we might be tempted to multiply these together to obtain 576 volt-amperes.

Referring to **Fig. 3**, we see the resolution of the currents and voltages. In equations (3-9) and (3-10), we resolve the current into the real part which is in phase with the line voltage and the imaginary part which is 90 degrees out of phase with the line voltage. At the bottom, we calculate the horsepower on the basis of 746 watts per horsepower. The power factor is simply the real power divided by the total volt-amperes.

The 0.074 horsepower does not seem to mesh very well with the 1/3 horsepower on the motor nameplate. We noted that the motor was idling. As you start to do some real drilling, say, using a half-inch drill in cast iron, the current creeps up slightly to 7.8 amps peak with a phase angle falling to 68 degrees. This gives a real current of:

i = .7071*7.8*cos(68)
i = 2.07 amperes RMS

Power is:
P = .7071*163*2.1 A
P = 238 watts = 0.319 horsepower

This is a little more in keeping with the 1/3 horsepower label. Note, however, that the Rochester Gas and Electric Company is supplying me with 7.8*115 = 899 volt-amperes, while the wattmeter is only billing me for 238 watts. There is nothing imaginary about the reactive volt-amperes. They heat transformers and wires just like the "real" power. The lamp draws leading current like a capacitor. This is because the lamp resistance goes up after the initial flow of current, so it tends to shut down before the peak of the cycle is reached. Because the phase angle is only a minus 17 degrees, the real part is:

irms = .7071*1.1*cos(17)
irms = .744 amps RMS

Power = 115.25*.744 = 85.72 watts

This is not so far off the target of the 100 watts listed on the bulb. Also, we used an RMS correction to apply to a distorted waveform. The power factor for the lamp is:

Power factor = 85.72/89.64 = 0.957

Much of the work to be done in impedance matching will be simply a matter of trying to correct the power factor of the load for efficient transfer of power.

Power factor correction

Let us suppose that we wanted to correct the power factor of the drill press. At no load, we see that the imaginary current is:

iimag = 0.707*7.07 amp*sin(84.5)
iimag= -j4.98 A RMS

Now, if we were to supply a capacitor that would draw +j4.98 A RMS, the capacitive current would cancel the inductive current and the power line input current would fall to 0.68 A RMS. From formula (2-18), we have:

iimag = V/Xc
Xc = 115.25/-j4.98 = -j 23.14 ohms

but

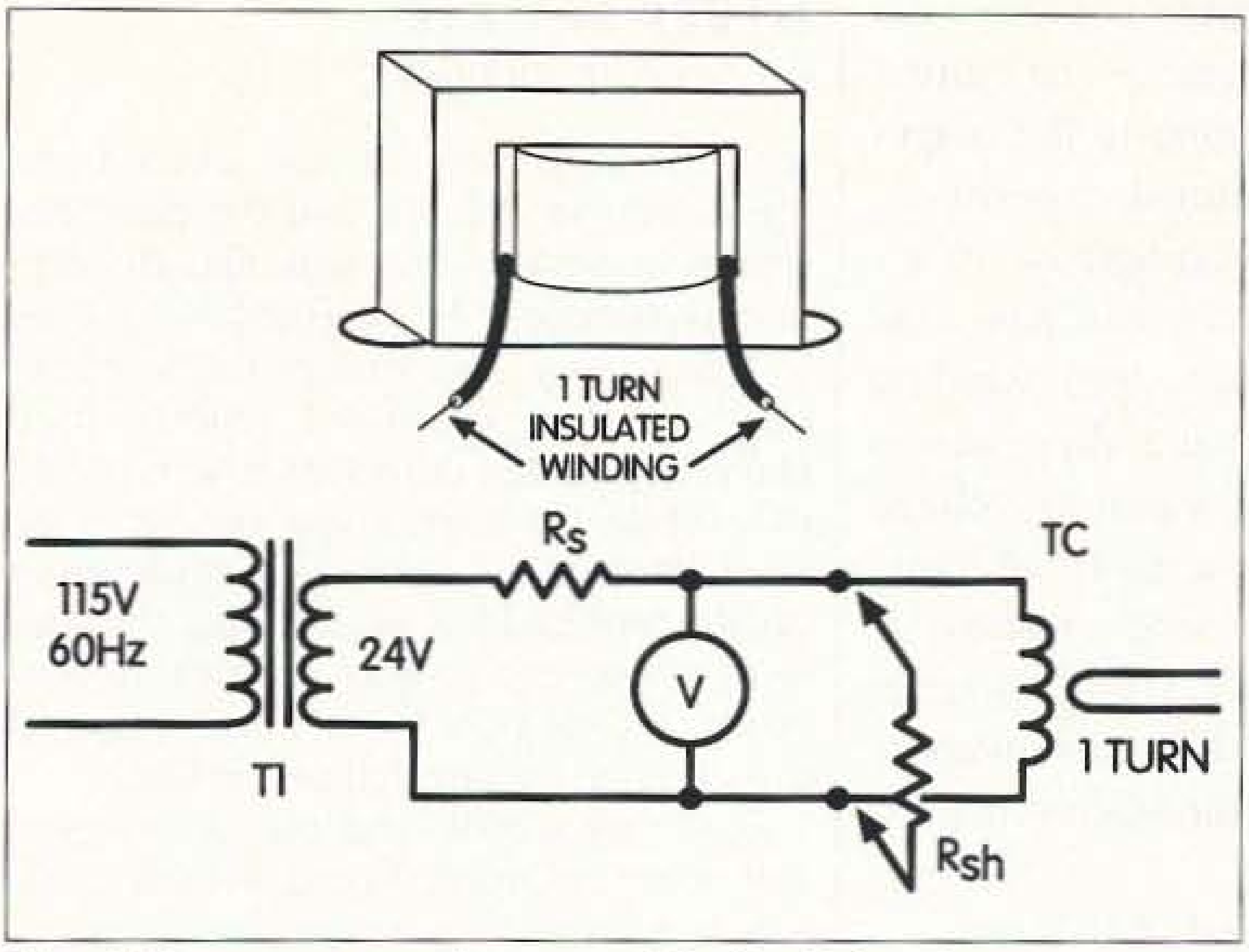

Fig. 4. The current transformer, and finding Rsh value.

$1/X_c$ = j0.0432 mhos = w*C = 2*pi*60*C

C = 0.0001146 Farads or 114 microfarads

This is a fairly large size for a capacitor that can handle 163 volts peak AC; therefore, power factor correction is rarely attempted on small machines.

Power companies can rely upon the diversity of their loads to smooth out the power factor, but a radio antenna system cannot. It must be impedance-matched if any reasonable transfer of power is to take place.

The current transformer

The current transformer shown in **Fig. 2** makes a worthwhile project for this chapter, and it is also a handy thing to have around the ham shack. Furthermore, it will teach us some valuable lessons about radio frequency measurements.

The current transformer can be made from nearly any transformer. For convenience it should be small, but the main requirement is that there be enough room to sneak a wire through between the winding and the core.

This is illustrated at the top of **Fig. 4**. The wire passes between the winding and the core, around the back, and out the other side. The wire should be insulated and of a size capable of handling the number of amperes you expect to measure. Do not use varnish-insulated wire, since the voltage rating of this wire is too low and the varnish may be scratched through when pulling the wire between the winding and the core.

Teflon-insulated hookup wire normally has a rating of 600 V if the insulation is about 1/32-inch thick. Remember that the voltage-to-ground of the circuit whose current is being measured will appear on this wire, and you may be handling the core and attaching the trans- former to a grounded oscilloscope.

As a simple guide to wire size, a #16 wire is safe at 15 amperes. If the space between the winding and the core is too small for this, two #18 or four #20 wires wired in parallel will also serve for 15 amperes. For other ratings, you can look up the area of the wire on a wire table and assume that you can run 1000 amperes per square inch of wire cross-section. This rating accounts for heating in the transformer and is on the conservative side.

The next thing to do is to find the correct value for a shunt resistor. At the bottom of **Fig. 4**, you will see a circuit hookup. The 24-volt transformer is used as a safety measure to isolate your setup from the power line. The specific voltage used is not important; however, 24 VAC is a safe level with which to work, and 24-volt transformers are widely available. Pick a value of Rs such that the voltmeter reading is about 10% of the T1 output voltage reading, with the circuit connected, except for Rsh. If we assume that the transformer you picked out for TC is a 115 V to 24 V variety rated at perhaps 1 A output, the value of Rs will work out to be about 10k to 12k ohms. The power being dissipated in this resistor will be somewhat less than 24*24/10000 = 0.056 watts.

Now what we need is to find a value

Continued on page 26

Secrets of Transmission Lines

continued from page 25

of Rsh that will reduce the voltmeter reading by a factor of about 10. It is important that transformer TC have no load on any secondary windings. To do this, simply clip various values of Rsh across the winding and observe the voltmeter reading. When the correct value of resistance is found, solder it in place across the winding of TC.

We have been talking about the fact that the voltage drop across an inductor is in quadrature with the current, and in fact, if the voltage drop across the winding of TC is 10% of the output voltage of T1, then the phase angle of the current in TC will be only 5.7 degrees.

Without belaboring the math too much, when the winding is shunted by Rsh such that it reduced the drop across TC by a factor of 10, an interesting thing happens. The ratio of the currents in the single turn winding and the current in the secondary is given by:

Ist/I2 = (j*w*M12)/(Rsh+j*w*L2)

where

Ist is the current in the single turn winding in

I2 is the current flowing through Rsh and L2

M12 is the mutual inductance between the windings

L2 is the self-inductance of the winding shunted by Rsh

Now, if w*L2 is greater than Rsh, then we may neglect Rsh. The j*w in the numerator and denominator will cancel, and:

Ist/I2 = M12/L2

There are a couple of important things here. First of all, we note that the currents are in phase. Secondly, we see that the ratio between the primary and secondary is independent of frequency — the j*w terms have canceled out. The output voltage will be:

Vout = I2*Rsh

Now, you will probably not know the value of M12, so the ratio of the current in the single turn winding to the output voltage is best determined experimentally. If you have or can borrow an accurate AC ammeter, you can pass currents through the single turn winding and measure the voltage drop across Rsh. If you have a variable voltage source like a variac or a multi-tap transformer, you can use a single resistor. If you have only a single voltage source, you can use resistors of different resistances to obtain several calibrating currents.

The power dissipated in Rsh is:
PD = (Vout*Vout)/Rsh

This can be substantial, and Rsh should have a wattage rating that is conservative. Note that if Rsh should open up or fall off, very high voltages can be generated in L2 or other windings on the transformer. Also note that any other secondary winding can have a substantial voltage on it.

Since the single turn winding must interrupt the circuit and power line voltages are liable to be found on it, it is well worth it to have sturdy terminals to attach to the single turn winding. I have found it convenient to place the transformer in or on a conventional electrical box, and to wire the single turn winding between a conventional outlet and a conventional plug. With this arrangement, an appliance can simply plug into the box, and the box can plug into a wall outlet for current measurement without cutting any wires.

If you have an oscilloscope, the current transformer can be used to show waveshapes and phase angles.

The cancellation of the j*w terms would imply that the frequency response might extend indefinitely. As a practical matter, the frequency response of the device is probably a function of the thickness of the core laminations. With standard 0.015-inch core laminations, the response will tend to fall off at frequencies in excess of 400 Hz or so. As we shall see later, a current transformer with a ferrite or powdered iron core is a significant part of most directional couplers, VSWR meters, and automatic tuners.

To be continued.