

Secrets of Transmission Lines

Part 2: Review of AC fundamentals.

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The original dynamo was an AC rather than a DC machine. However, alternating current seemed to be significantly less useful than direct current. One cannot charge a battery, conduct electrolysis, or electroplate with alternating current. Arc lights operate poorly on AC and welding is rendered more difficult. Furthermore, the behavior of circuits excited with alternating current was profoundly more difficult for investigators in the early 1800s to understand.

Probably the most puzzling item was the fact that Kirchhoff's Law was not obeyed.

In the circuit similar to Fig. 3 of Part 1 and equation 1-6, the value of i was not necessarily equal to the sum of $i_1 + i_2 + \dots + i_n$; in fact, it was usually smaller. In a series circuit, E was not necessarily equal to $E_1 + E_2 + \dots + E_n$; in fact, it was usually smaller, too. In addition, current would seem to flow in circuits where there was no connection. It is not hard to imagine that it would take a considerable amount of investigation to explain this behavior.

In 1820, Hans Christian Oersted found that an electric current flowing in a wire would deflect a compass. In the same year, François Arago found

that a helical coil of copper wire, otherwise nonmagnetic, would behave identically to a bar magnet when an electric current flowed through it. This work attracted the attention of André-Marie Ampère, who showed that there is a physical attraction between wires carrying currents flowing in the same direction and a repulsion between wire carrying currents in opposite directions. This is very easy to prove experimentally. The apparatus of Fig. 1 is easily made from a couple of screw eyes and a length of stiff copper wire. The stronger the current, the farther the bail will swing out.

Note that the battery polarity can be reversed without changing the result, since the current in both the stationary loop and the bail are reversed. A small transformer with a lamp in series to limit the current can be substituted for the battery and will also do the same thing. The angle of the bail will be proportional to the absolute value of the average current. By absolute value we mean without regard to algebraic sign. If the algebraic sign is included, of course, the average current is zero since you have as many positive as negative cycles.

Electromagnetic induction

In 1829, Joseph Henry wound a layer of insulated wire on a U-shaped iron core and found that it made a very powerful magnet when energized. The illustration in Fig. 2 shows the electromagnet with a significant addition. Henry found that when the switch was closed, there was little or no sparking; however, when the switch was opened, there was a large and vigorous spark—much more energetic than could be obtained from the battery alone. In 1831, Michael Faraday found that a second winding not electrically connected to the first would show an electrical impulse when the battery was first connected and also when the circuit was opened. This is termed electromagnetic induction. A changing current in the primary circuit induces a voltage in the secondary

To duplicate the experiment, it is not necessary to wind a lot of wire, although this could be done using a large iron nail. Small, cheap transformers (for example, 12 V, 300 mA output) are often made with the "E" and "I" core pieces not interleaved. If you pry open the metal frame, the "I" will fall off in one piece. With a single D cell,

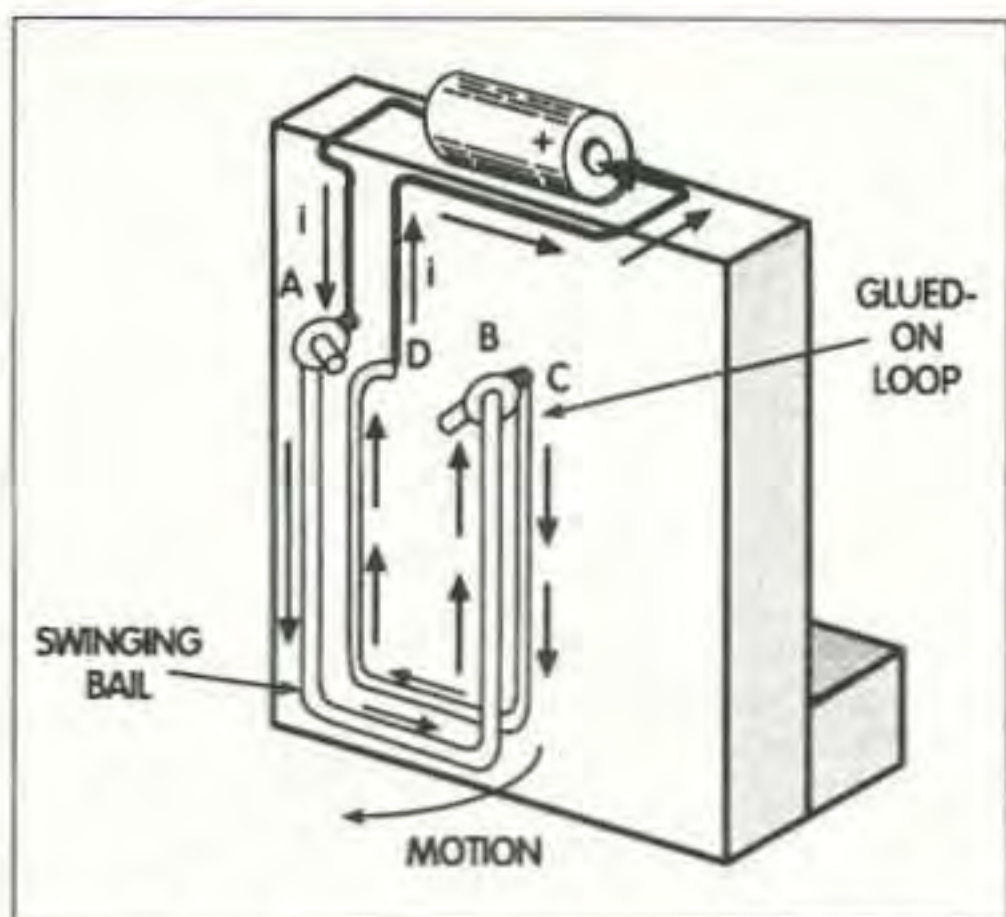


Fig. 1. When the circuit is closed, a "D" cell will drive about an ampere through the circuit and the bail will kick out sharply.

you can investigate the magnet properties and the primary sparking. With an analog voltmeter on the other winding, you can investigate the induction. The analog meter is specified because it shows transient voltages better. You can see how far the needle kicks, whereas a digital meter just flashes digits too fast to follow.

You really do not need a switch. You can open and close the circuit just by touching the wire to the battery. However, if you are holding both sides of the circuit when you break it, you can get a rousing shock even with just the 1.5 volt battery. The spark voltage is many times higher. We will get back to this shortly.

Capacitance

Another property to be investigated is capacitance. When people were working with static electricity, a standard

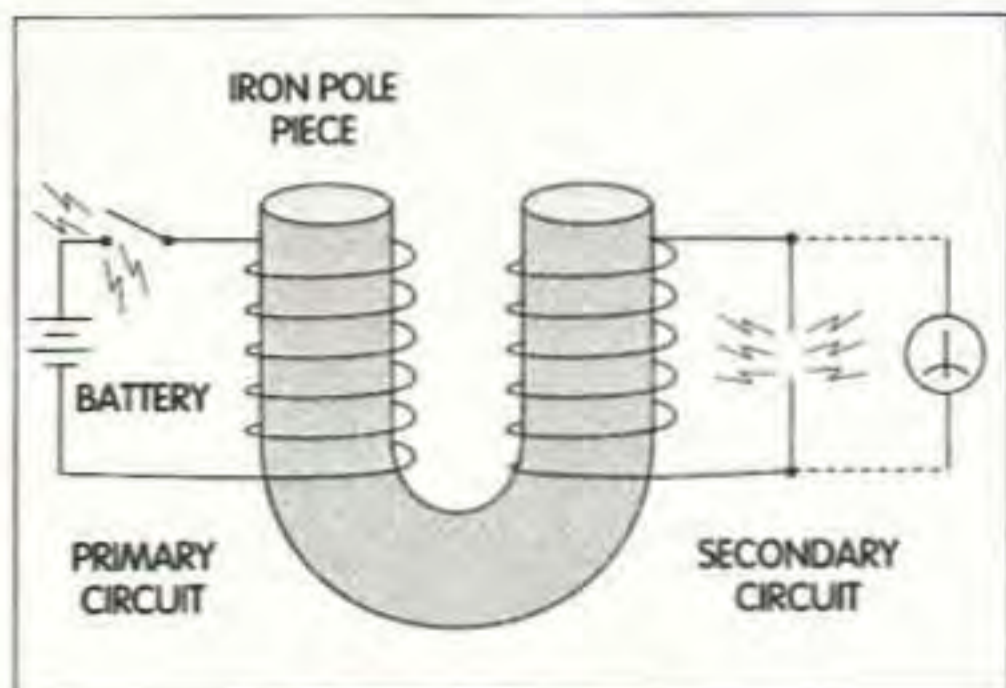


Fig. 2. The electromagnet and electromagnetic induction. When the current is established in the primary circuit, energy is stored in the magnetic field. When the current is interrupted, this energy is dissipated in the form of one or more arcs.

device used was the "condenser" or Leyden jar. The Leyden jar was a glass jar with foil on the outside and the inside with no electrical connection between them. There was usually an insulating cork with a rod sticking through that contacted the inside foil. With an electrostatic generator you could "charge up" the jar—and the spark that ensued when it was discharged was much more vigorous than what could be obtained from the electrostatic generator alone.

As an experiment, take a 9-volt transistor battery, an analog voltmeter with a 10 or 12 volt scale, and an electrolytic capacitor with a 1000 or 2000 μF rating (of course, with a 9 or more volt rating). Touch one of the capacitor leads to the battery, and connect the other to the battery through the voltmeter, observing polarity. The voltmeter will initially jump to a 9 volt reading (it may even overshoot a bit) and then the voltage reading will taper off, eventually winding up at zero. The capacitor is now charged, and its voltage is equal to the battery voltage so the voltmeter reads zero.

Short the capacitor with something, and you will see a substantial spark. If you short the capacitor on a piece of printed circuit board, it may even blow a hole in the foil. Note that shorting the battery itself will not make a visible spark. The capacitor stored energy from the battery over a period of time and released it in a much shorter period—therefore the vigorous spark.

Stored energy

In both the inductor and the capacitor, the energy is taken up relatively slowly and the spark is evidence of the sudden release of the stored energy. In the inductor, the energy is stored in building the magnetic field. This is a form of kinetic energy and quite analogous to the action of a hammer. You store energy of motion swinging the hammer, and the hammer imparts the energy to a nail in the few milliseconds that it takes for the nail to stop the hammer head. The energy stored in the inductor is:

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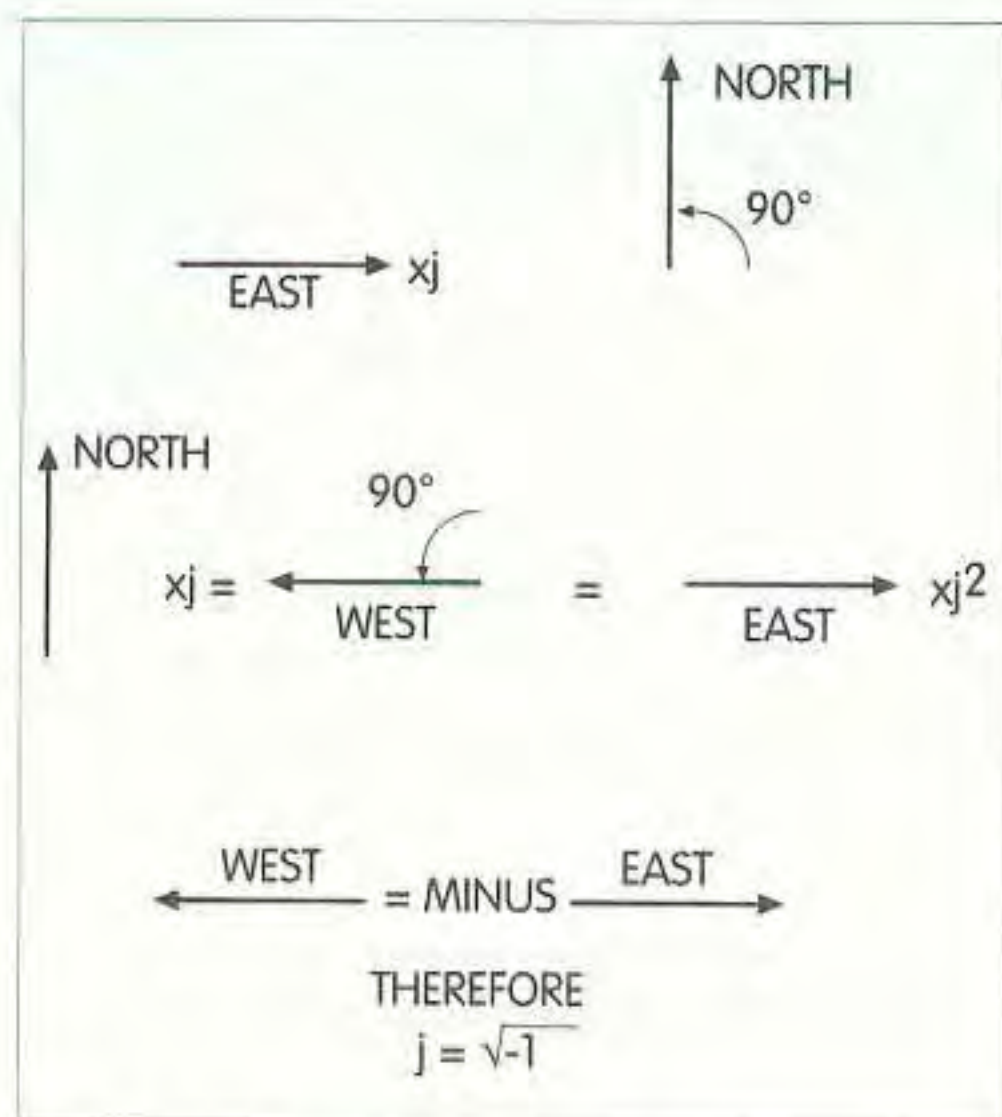


Fig. 3. The imaginary operator "j".

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$$W = (L \cdot i^2) / 2 \text{ joules or watt seconds} \\ \text{eqn (2-1)}$$

where

L is inductance in henrys
i is current in amperes

If the inductance had been large enough and you measured the current when the connection was first made, you would see that the current started at zero and built up slowly. The rate at which the current builds is:

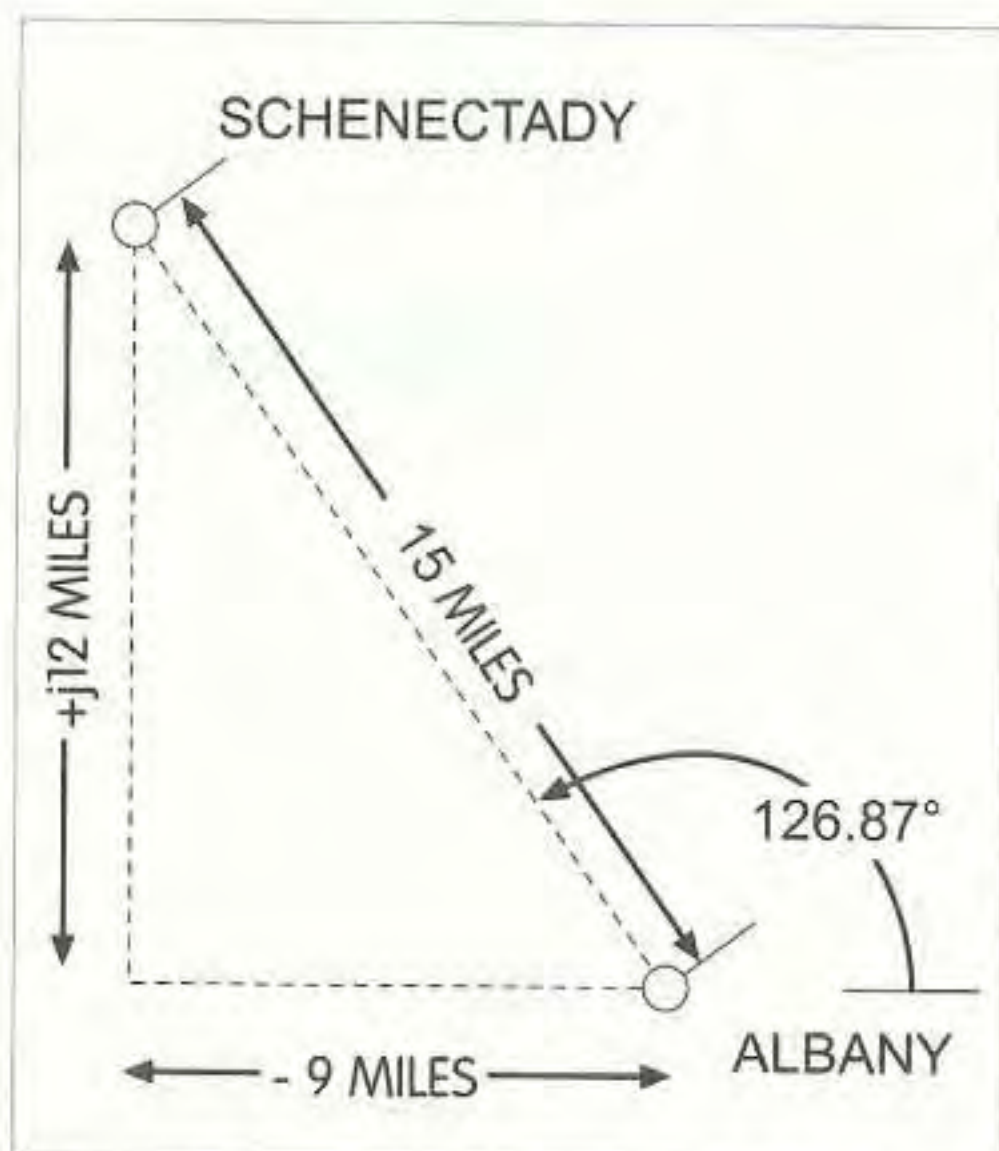


Fig. 4. The vector distance from Albany to Schenectady. If we define the plus direction as east and the +j direction as north, then the minus direction is west and the -j direction is south. The path, or vector distance, is $-9 + j12$ miles, or $15/126.87^\circ$ miles.

$$d_i/d_t = E/L \text{ amperes per second} \\ \text{(2-2)}$$

where

E = volts

L is inductance in henrys

On the other hand, the energy stored in the capacitor is potential energy. It is analogous to the energy stored in a spring when it is compressed or stretched. The energy in the capacitor is:

$$W = (C \cdot E^2) / 2 \text{ joules} \\ \text{(2-3)}$$

where

C is capacitance in farads

The voltage across the capacitor is given by:

$$E = Q/C \text{ volts} \\ \text{(2-4)}$$

where

Q = charge in coulombs

but Q is equal to the integral of $i \cdot dt$.

A little AC math

In order to go through some of the subsequent explanations, it becomes necessary to resort to some vector algebra, which sounds a little more frightening than it actually is. Suppose that you tell the air traffic controller that you are departing at 200 mph. You haven't told him where to find you. At the end of an hour you could be anywhere on a circle of 200 mile radius. However, if you tell him that you are departing at 200 mph on a course of 90 degrees, he will know that at the end of an hour he will find you at a point 200 miles to the east.

The latter is a *vector* quantity. You know where it starts, how fast it is going, and in which direction. Quantities that have no direction—such as dollars, watts, temperature, and population—are called *scalars*.

One of the first things we have to find out about is the "imaginary" operator "j". Suppose that we wish to assign an operator that will rotate a vector 90 degrees in a counterclockwise

direction. Referring to Fig. 3, let us assume that we have a vector one unit long pointing east. If we apply the operator "j" once, the vector is now north. Applying the operator a second time leaves the vector west. But west is equal to minus east. Therefore, $j \cdot j$ or j^2 is equal to a minus one and $j = \sqrt{-1}$ —and there is no such number!

Charles Proteus Steinmetz worked in the General Electric laboratories in Schenectady, New York. He was fond of saying that there is nothing more imaginary about imaginary numbers than there was about the distance between Albany and Schenectady.

Referring to Fig. 4, and defining + as east and +j as north, we see that we could describe the distance as $-9 + j12$ miles.

Further, using the Pythagorean theorem we can compute the straight-line distance as:

$$\text{dist} = \sqrt{[(9 \cdot 9) + (12 \cdot 12)]} = 15 \\ \text{miles}$$

and

$$\text{angle} = 180 - \arctan(12/9) = 180 - 53.13 = 126.87 \text{ degrees}$$

The first of these descriptions is in rectangular or Cartesian coordinates and the second with a length and an angle is in polar coordinates. When vectors have to be added or subtracted, they are most easily done in rectangular coordinates. Vector multiplication and division are most easily done in polar coordinates.

Another useful relationship is Euler's equation. When we describe a vector such as the one above as 15 miles/126.87 degrees we see that this is really a shorthand way of describing Euler's equation as shown in Fig. 5. Euler's equation is a way of transforming between coordinate systems.

Back to components

Now let us consider some AC cases. Our instantaneous AC voltage is described as:

$$V = V_0 \cdot \sin(\omega \cdot t) \text{ volts} \\ \text{(2-8)}$$

where

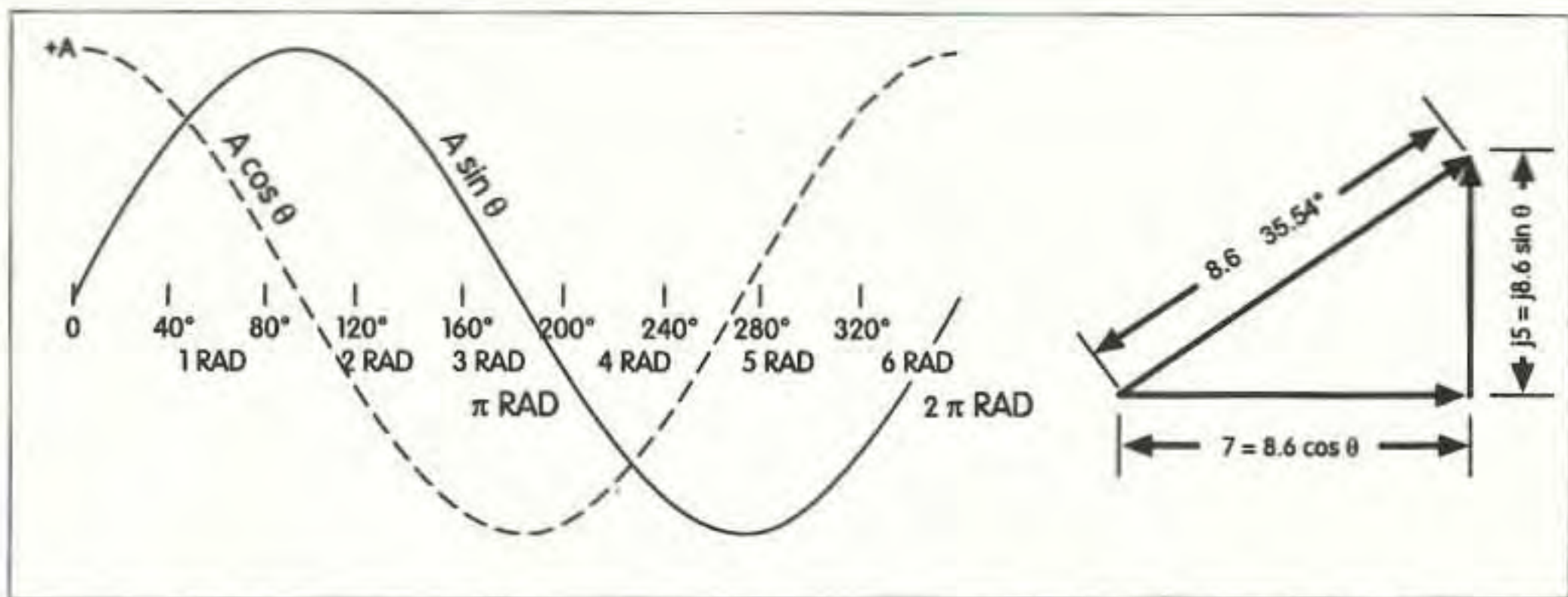


Fig. 5. Euler's equation. The symbol e represents the natural logarithm base, 2.718. The term $Ae^{j\theta}$ describes the location of the tip of a vector of length A rotated through an angle θ .

$$(2-5) Ae^{j\theta} = A \cos \theta + jA \sin \theta$$

$$(2-6) Ae^{-j\theta} = A \cos \theta - jA \sin \theta$$

$$(2-7) 8.6 \angle 35.54^\circ = 8.6e^{j35.54^\circ}$$

V_o is the peak AC voltage
 ω is angular frequency $2\pi f$ in radians per second
 f is cycles per second, or hertz
 t is time in seconds

Note that we could have written this:

$$V = V_o e^{j\omega t} \quad (2-9)$$

And for the inductor, we noted in eqn (2-2) that:

$$d_i/d_t = E/L$$

and, rearranging,

$$V = L(d_i/d_t) \quad (2-10)$$

Note here that we have substituted the instantaneous voltage, and from (2-8):

$$V_o \sin(\omega t) = L(d_i/d_t)$$

and, rearranging,

$$d_i = (V_o/L) \sin(\omega t) d_t$$

Integrating gives

$$i = -[V_o/(\omega L)] \cos(\omega t) + \text{constant} \quad (2-11)$$

The constant can be neglected because it pertains to transient conditions and we are concerned only with steady-state conditions here.

Now, from the curves of Fig. 5, we see that:

$$-V_o \cos(\omega t) = V_o \sin[(\omega t) - 90] \quad (2-12)$$

Thus,

$$i = [V_o/(\omega L)] \sin[(\omega t) - 90] \quad (2-13)$$

What this tells us is that the inductor current lags the applied voltage by 90 degrees. In most AC problems we are interested in the average results over many cycles, and so we carry the (ωt) term implicitly and simply neglect to write it down. Also, we learned that the operator "j" would rotate the vector by 90 degrees. Making use of both of these conventions, we may rewrite (2-13) as:

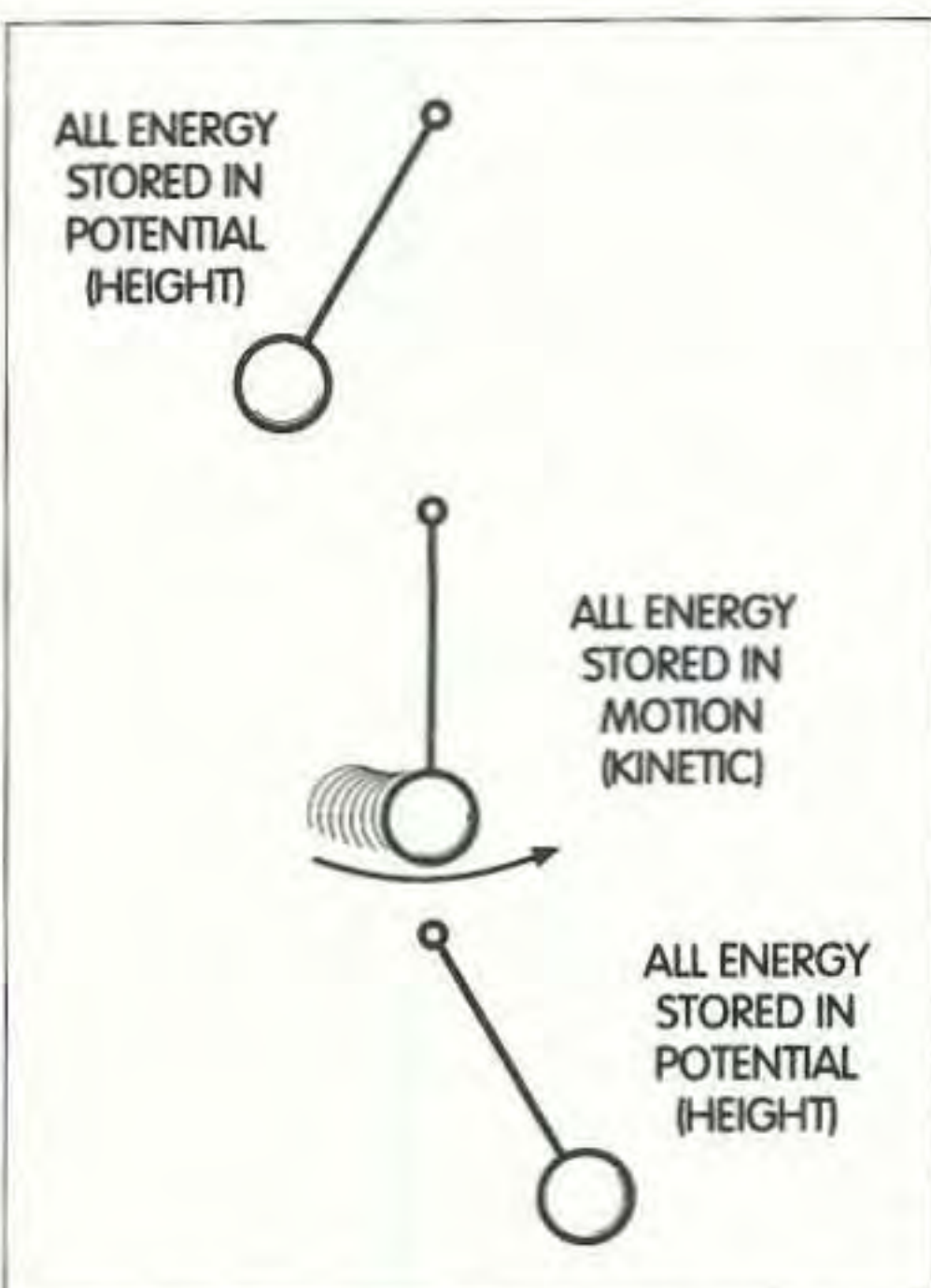


Fig. 6. The pendulum.

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$$i = [V_0 / (j * \omega * L)] \text{ amperes} \\ (2-14)$$

The term $(j * \omega * L)$ is called the *inductive reactance*. It describes the opposition to AC current flow just as resistance does for DC in Ohm's law.

In practical inductors, there is always a resistance in the circuit too. Since the resistor tends to draw current in phase with the voltage, the terms combine as follows:

$$i = V / [R + (j * \omega * L)] \\ (2-15)$$

The term $[R + (j * \omega * L)]$ is called *impedance*. Impedance plays the same role in relating voltage and current for AC as resistance plays for DC. We will see shortly that capacitance may also enter into impedance.

For the capacitor, we noted that:

$$V = Q / C \text{ volts}$$

where

Q = the integral of current with respect to time, measured in coulombs or ampere seconds

If the capacitor drew an average of one ampere in the first second, a half ampere in the second, and a quarter ampere in the third and so forth, then Q would be equal to $1 + 1/2 + 1/4 + \dots$

Now, applying an AC voltage, we obtain:

$$V_0 * \sin(\omega * t) = Q / C \\ (2-16)$$

Differentiating the above expression with respect to t , we obtain:

$$d_Q / d_t = [V_0 * \omega * C * \cos(\omega * t)] \text{ amperes} \\ (2-17)$$

Note that if Q is the integral of current with respect to time, then dQ/dt is current.

Taking again the liberty that for most problems we are interested only in long-term averages we may write that for a capacitor:

$$i = V/[-1/(j\omega C)] \text{ amperes} \\ (2-18)$$

Note the similarity and the differences of the result for the inductor and the capacitor. Whereas the instantaneous current in the inductor is given by $-\cos(\omega t)$, the instantaneous current for the capacitor is given by $+\cos(\omega t)$. Also, where the inductive reactance is given by $(j\omega L)$, the capacitive reactance is given by $-1/(j\omega C)$. We also see that inductive reactance is directly proportional to inductance, whereas capacitive reactance is inversely proportional to capacitance. The reactances are usually referred to as X_L and X_C respectively.

An interesting case occurs in the event that $X_L = -X_C$. If the two elements are connected in parallel and both are theoretically perfect with no losses, it takes no current on the main line to excite a large current in both elements. The case is similar to the pendulum shown in **Fig. 6**.

At the end of the swing, all of the energy is stored in the form of potential energy like a capacitor. At the bottom of the swing, all of the energy is stored in the motion of the bob, like the magnetic field of the inductor. As the bob continues its swing to the other end of the travel, all of the energy is converted again into potential energy in the capacitor.

In the parallel circuit the energy simply sloshes back and forth between the inductor and the capacitor. If no losses existed in the circuit, and the pendulum dissipated no power, both would continue forever!

Conclusion

Having seen some of the characteristics of reactive elements, next time we'll deal with some real circuits and discuss power factor, power dissipation in an alternating current circuit, and the difference between kilowatts and kilovars. 