How Many Kinds Are There?

By "CATHODE RAY"

THIS symbol (presumably short for "quality factor") has become generally accepted as the prime virtue where r.f. components are concerned. It has even been incorporated in trade names. So recent statements that Q-meters don't read Q may have sounded to some like a tampering with the eternal verities.

What exactly is Q? Although it has been in common use for so long it has been slow to be officially recognised. Perhaps that is because a thing that has gone about with several different meanings seems hardly respectable in official circles.

Its roots lie in the early days of broadcasting, when transmitters were low-powered and none too easily heard with the single-valve or crystal receivers of that period. So the demand was for tuning coils that would make the most of the feeble r.f. voltages picked up. Next, when stations multiplied in number and power, the problem was not so much to tune in the wanted station as to tune out the unwanted ones. All this time the wireless amateurs' papers were full of advice on coils—practical advice on how to wind better coils, and theoretical advice on the underlying principles. It was shown that the coil which could give the strongest signals was also the most selective (though the optimum tapping or coupling depended on which quality was needed most).

The first prescription for achieving this double benefit was to reduce the r.f. resistance as much as possible. While quite true so far as it went, this was not the whole truth—it was soon realised that coil A might have a lower r.f. resistance than coil B and yet be less efficient in the two essential respects of sensitivity and selectivity. A resistance of 20 ohms would be bad in a medium-wave coil, but good in a long-wave coil. To make a fair comparison one had to take into account their inductances, and the frequencies at which they were used. So the need was felt for a single figure that would include all the factors concerned.

As a matter of general principle a standard of goodness, or a "figure of merit," is preferable to a standard of badness such as r.f. resistance.

That was where the term "circuit magnification" or "magnification factor" (abbreviation "m") came in.* It was based quite simply on the elementary principle of resonance, as shown in Fig. 1. If the frequency of the "input" or series voltage, v, is adjusted to make the reactances of L and C equal, they cancel one another out, leaving R as the sole impedance of the circuit so far as v is concerned. The current is therefor equal to v/R. But this current flows through C and L, and sets up voltages across them, equal to the current multiplied by their reactance. As the reactances are equal and the current is the same, the voltages are equal, and can both be denoted by V. Reckoning from the inductive reactance, $2\pi f$L (abbreviated to ωL), we have:

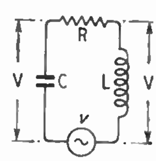


Fig. 1. Simple resonant circuit, with injected voltage v. The resulting voltages across C and L are in opposite phase and (at resonance) equal, so that reactances of C and L cancel one another out and the strength of current in the circuit depends only on R.

$$V = \frac{v\omega L}{R}$$

The interesting thing, of course, is the ratio of V to v, because V is the "output" voltage, which can be used or passed on to the next stage; v being the input, derived perhaps from an aerial or a valve coupled by a primary winding. In any reasonable tuning circuit V is considerably greater than v, so it was natural to call V/v the magnification. We have, then:

$$m = \frac{V}{v} = \frac{\omega L}{R}$$

If we reckon from the capacitive reactance, $1/\omega C$, we get $m = \frac{1}{\omega CR}$, which comes to the same thing—in Fig. 1, at least.

Instead of approaching the matter in this theoretical way, one may prefer to inject an actual voltage into a real tuned circuit and measure the output voltage across it; m is then directly $\frac{V}{v}$.

In the course of time the Americans, thinking on similar lines, began to use the expression "Q." As it was usually defined as $\frac{\omega L}{R}$, it was generally assumed to be another name for "m," which it has tended to oust. But some slightly different definitions of Q appeared from time to time; and in the absence of prompt and firm action by acceptable authority, a state of uncertainty set in, and the term "Q" was generally avoided by the most precise people. Everybody else, however, found it too convenient for such scruples to prevail, and a Q-meter became one of the most used tools in almost every radio laboratory, while lots of people who hadn't the least idea what it really meant discovered in Q a

* As far as I have been able to trace, the earliest use of voltage magnification as a standard of coil efficiency was made by S. Butterworth (*Experimental Wireless and Wireless Engineer*, May, 1926, p. 267).

Q—

valuable addition to their sales talk.

Many people in the radio business can get along quite well with the single easily-absorbed fact that a high Q means good selectivity and signal amplification. That is the great merit of the expression; it means something in terms of practical results. One does not need a university education to grasp its general significance. I take it, however, that if you were content with rough ideas you wouldn't be reading this; so we will now proceed to consider the meaning of Q in greater detail.

Most of the controversy on the subject arises from the fact that no actual circuit is so simple as Fig. 1. L, C and R are shown there as separate components, but of course that is a theoretical simplification. R represents the total of the various forms of resistance and loss throughout the circuit. Normally most of it is the resistance of the coil, so L and R together are often assumed to represent the coil: but the capacitor is bound to have some resistance, so for more exact analysis one would divide R into two portions, attached to L and C respectively. We shall see later that if R is not substantially smaller than ωL and $1/\omega C$ it is necessary to be particularly careful how *m* and Q are defined or measured.

Other complications occur because in practical circuits the

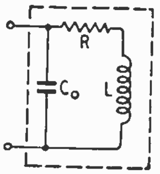


Fig. 2. A tuning coil can be represented fairly accurately by this equivalent circuit.

capacitor contains some inductance, and the coil contains some capacitance. The inductance of a well-designed capacitor is usually negligible except at very high frequencies; but the self-capacitance of a coil is by no means negligible, and is responsible for the largest discrepancies between different ways of arriving at Q. For one thing, as we shall see, it raises questions about how the input voltage v is brought into the circuit.

At very high frequencies there is not even an appearance of L and C being separate—the tuning circuits are composed of parallel

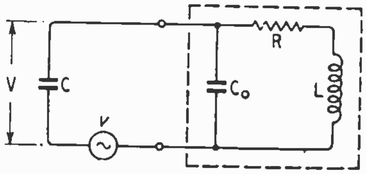


Fig. 3. When a real coil is substituted for L in Fig. 1, the "coil equivalent circuit" of Fig. 2 shows that the complete circuit is not quite the same, and measurements based on the assumption that it is will be wrong.

rods or cylinders, or of hollow spaces, in which L and C are inextricably mixed up and distributed. What about Q then?

We shall leave that question until later, and assume first that the frequency is moderate enough to let us represent the actual tuned circuit reasonably accurately by a diagram made up of separate lumps of L, C and R. That being so, it is usually satisfactory to consider the coil as if it were composed as shown in Fig. 2, in which C_0 is the self-capacitance.

Comparing this with Fig. 1 we see that the coil is itself a complete resonant circuit. It is not possible to open the circuit to insert a signal source directly in series as in Fig. 1—the dotted line is a reminder that the items within it are only theoretically separable—but its equivalent can be performed by inductive coupling. The frequency at which a coil resonates on its own is called the self-resonant frequency. Although coils (especially if permeability-tuned) can be employed in this fashion, it is unusual to do so, because it allows the resonant frequency to be affected so much by stray capacitance. Nearly always the coil is used with a separate tuning capacitance.

Although the r.f. resistance of a capacitor can be kept very much smaller than that of a coil, it may not always be negligible. So it is necessary to make quite clear whether one is considering the Q of the coil alone, of the capacitor alone, or of the whole circuit. Just now we shall assume that the capacitor is perfect (zero resistance; infinite Q), so the Q of the coil is the same as the Q of the circuit.

Assuming also that the voltage v is introduced in series with L (in practice, by inductive coupling) connecting a perfect tuning capacitor across the terminals in Fig. 2 makes no difference in principle. It comes directly in parallel with C_0, and for purposes of calculation two capacitances in parallel can always be replaced by one equal to their combined values; so the actual circuit is unchanged. But if the signal source is connected in series with the *coil* (which is not just L, but the whole combination inside the dotted line), we have a different circuit arrangement, Fig. 3. The question then arises; are we concerned with the true inductance of the coil (L) or the inductance as it appears to be at that particular frequency, supposing that the

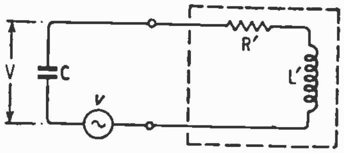


Fig. 4. At any particular frequency the coil equivalent circuit can be replaced by an apparent inductance and apparent resistance in series, which enables the principles of Fig. 1 to be applied.

dotted line contained only inductance and resistance is in Fig. 4? The apparent inductance (L′) is not quite the same as L—it must be greater, to make up for ignoring C_0—and R′ is not the same as R. If they both differed in the same ratio, then the value of Q (taking it to be $\omega L/R$) would be unaffected, but as it happens they are not. The textbooks show that

$$R' = R\left(\frac{C + C_0}{C}\right)^2$$

and

$$L' = L\left(\frac{C + C_0}{C}\right)$$

so what we may call the apparent Q, denoted by Q′ and equal to $\omega L'/R'$, is

$$Q' = \frac{\omega L'}{R'} = \frac{\omega L}{R}\left(\frac{C}{C+C_0}\right) = Q\left(\frac{C}{C+C_0}\right).$$

When the external tuning capacitance C is very much larger than the self-capacitance C_0 the difference between Q and Q′ is not worth bothering about. A typical self-capacitance is 6pF, and if the added capacitance were, say, 300pF, $Q' = \frac{300}{306}Q = 0.98Q$; the difference would be only 2%, which is less than the probable error of most Q-meters. But if no C is used the apparent Q is zero, no matter how high the true Q may be! So the distinction ought not to be completely ignored.

Opinions have differed as to which Q is the right one, or in fact whether either as defined above is right. To settle the question some people appeal to basic principles and others to practical sense. To serve its purpose of expressing the goodness of a tuning circuit or component it would obviously be a great advantage if the definition corresponded to the method of use. So we had better consider how tuning circuits are used.

In a typical broadcast receiver there are three main kinds of tuned circuits, shown in rough outline in Fig. 5. There is first the r.f. circuit, L_1C_1, into which the input voltage is inductively injected from the aerial, and the output taken from across C_1. Next there is the i.f. primary, in which the mode of operation is reversed; the input is received directly across the terminals of C_2 and the output is imparted inductively, proportionately to the current flowing in L_2. Lastly the secondary, L_3C_3, which works similarly to L_1C_1.

None of these tuned circuits corresponds to Fig. 3; in all of them the self-capacitance of the coil is effectively in parallel with the external tuning capacitance, making a total of $C + C_0$, tuned by the true inductance L and damped by the true r.f. resistance R. There is no need to bother about L′ or R′—or Q′. The typical examples just shown cover the vast majority of tuned circuits in actual use. It is clear then that Q corresponds to practical affairs more closely and more often than Q′.

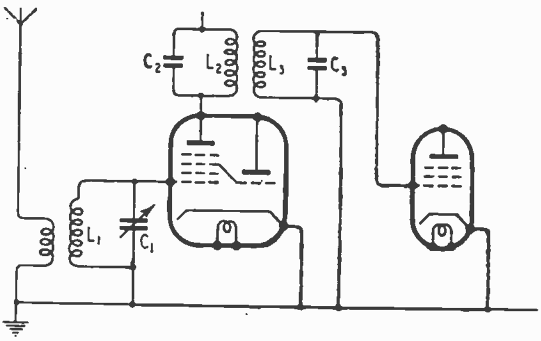


Fig. 5. Three typical ways in which tuning circuits are used in a broadcast receiver. In all of them it is true Q that counts, rather than the apparent Q.

But what about the methods used for measurement? The bare bones of the usual type of Q-meter are shown in Fig. 6. A variable-frequency oscillator is provided to pass a measurable current (I) through a known low resistance r. The r.f. voltage developed across r is therefore Ir, and it corresponds to the signal source in Fig. 3. The output voltage V is measured by a valve voltmeter across C, when C or the frequency of the oscillator has been adjusted to cause resonance, indicated by maximum V.

We must conclude, then, that the quantity which applies to the commonest methods of use is Q, but that the quantity actually measured by the commonest method is Q′. And therefore that when these methods giving Q′ are used, the readings should be multiplied by $\frac{C + C_0}{C}$ to bring them to Q. The instruments are, or should be, calibrated in C, and can be used to measure C_0. As we have already seen, the correction is hardly worth applying when C is many times greater than C_0; but omitting to apply it when C is not much greater than C_0 gives results which differ largely from the true Q.

A Q-meter is very handy to use, but is subject to another error—serious at the higher radio frequencies—due to r, which makes the instrument read lower than it should by increasing the resistance of the circuit being tested. Even if r were directly in series with R, so that it could just be deducted from it, one would have to calculate R, which is a nuisance with an expensive instrument that is supposed to read Q directly without any need for calculation. But actually r is in series with R′, so to be strictly correct one would

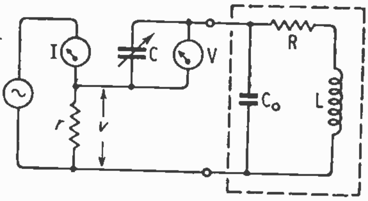


Fig. 6. Outline circuit showing the principle on which most Q-meters work.

have to apply the factor relating R to R′. In fairness to Q-meters I must admit that r is usually small enough to be neglected except in high-Q, very-high-frequency circuits, and also that some Q-meters work on different principles. When measuring very good coils one might also have to allow for the losses due to the valve voltmeter and the tuning capacitor. So it is as well not to be too impressed by the apparent direct-readingness of an instrument having a pointer moving over a scale marked "Q." Its great advantage is that it does give quite quickly and easily a figure that can be used for comparing one coil with another, even though that figure may often differ appreciably from the true Q. The instrument can also be used for a variety of other measurements if one is prepared to do a few simple calculations.

But if one is prepared for that there is a lot to be said for an alternative method—the method in which the frequency of the oscillator is read at resonance and also at the two settings, one on each side of resonance, at which the voltage across the tuned circuit is 70.7% (i.e., $1/\sqrt{2}$) of its maximum reading (Fig. 7). Then if f_r is the resonant frequency and f_1 and f_2 respectively the higher and lower of the other two:

$$Q = \frac{f_r}{f_1 - f_2}$$

In this method, the oscillator

Q—

is loosely coupled to the coil under test; there is no need for the r.f. ammeter or the resistance r; the result is given directly in true Q; and the method can be used in circumstances where the Q-meter fails. And of course it is very much cheaper.

The reason why it gives true Q is that the input voltage is inductively coupled to the coil under test, so is in series with the tuned circuit as a whole. In Fig. 6, by contrast, the input voltage is in series with only one of the two capacitance branches; C_0 forming a sort of bypass.

There is another feature about Fig. 6, which is of practical importance only when Q is exceptionally low, but is interesting theoretically. We have not defined "magnification factor," and I have yet to come across a really water-tight definition, but it seems to be generally agreed that it is V/v in Fig. 4 when the circuit is at resonance, as indicated by a maximum reading of V. If you ask whether this is not identical with what we have been calling Q', the answer is—not exactly. If you look up any good textbook that deals with resonance you will see that the frequency at which the voltage across the resonant circuit is maximum is not quite the same as the frequency giving series resonance. As a matter of fact, it depends on whether the maximum is arrived at by adjusting the frequency or by adjusting the tuning capacitance. Now Q (and Q'), as we saw in connection with Fig. 1, are based on the theory of series resonance. But Q-meters, which are the practical embodiments of Fig. 4, are so arranged that resonance is judged by the maximum reading of V. So really they are magnification-factor meters.

The relationship between m and Q' can be worked out. The calculation is rather involved, but as a matter of interest the result, assuming resonance is obtained by varying the frequency of the oscillator, is:

$$Q' = \sqrt{\frac{m^2 - 1 + m\sqrt{(m^2 - 1)}}{2}}$$

For example, if $m = 2$, $Q' = 1.8$—a 10% discrepancy; but if $m = 10$, $Q' = 9.96$—only 0.4% different.

If resonance is obtained by varying C:

$$Q' = \sqrt{(m^2 - 1)}$$

The discrepancy is slightly larger in this case, but is still utterly negligible for normal tuning circuits. It should not be forgotten when dealing with very "flat" circuits, however.

In the alternative (Fig. 7) method, too, resonance is judged by maximum V; but the resulting error is even smaller than in the previous cases. The calculation

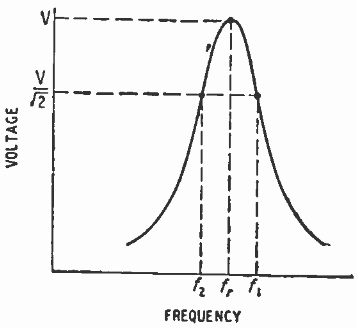


Fig. 7. Principle of an alternative method of Q measurement

is still more complicated, and the final result too bulky to be worth printing here.

The fact that Q is a measure of the effectiveness of a tuning circuit as regards signal strength is brought out obviously enough in the Q-meter method. The frequency-variation method, on the other hand, brings out the relationship of Q to selectivity. What one does, in fact, is to measure the sharpness of resonance. But that is not quite the same thing as selectivity in its most directly-useful terms. What we generally want to know is the bandwidth, in cycles per second, irrespective of the mean or carrier frequency. To tune in Droitwich, f_r in Fig. 7 would have to be 200 kc/s, and one might decide that the 70% points (f_1 and f_2) of a single tuning circuit ought to be, say, 6 kc/s above and below f_r, giving a bandwidth of 12 kc/s. The required Q would therefore be $\frac{200}{12} = 16\frac{2}{3}$. For tuning-in a station on 1200 kc/s with the same selectivity, the bandwidth would still have to be 12 kc/s, but the Q to give that selectivity would be $\frac{1200}{12} = 100$.

For constant selectivity, then, Q has to be proportional to frequency; so the quantity that indicates narrowness of bandwidth is not $Q = \frac{\omega L}{R}$, but $\frac{L}{R}$, the "time constant." At any given frequency, however, it is true to say that selectivity is directly proportional to Q.

This may be a good moment at which to point out another advantage of Q as a standard, compared with R. We have already seen that it is a fairer guide to the effectiveness of a coil because it takes into account its inductance, and also it is a measure of goodness rather than badness, and directly tells one the output voltage produced at resonance by a given input voltage. The other thing is that R, unlike ordinary d.c. resistance, is by no means constant. Most of the losses included in it tend to increase with frequency. Over a limited range of frequency, such as that covered by a tuning coil, the resistance R is usually roughly proportional to frequency. So, since $Q = \frac{2\pi f L}{R}$, over the same range of frequency Q is roughly constant. Only roughly; but at least it is more nearly constant than R.

So far we have been considering Q as a property of a coil, which is the same thing as the property of the whole tuned circuit, if losses outside the coil are negligible. But one often sees references to the Q of a capacitor or other component. The same principle holds: it is the ratio of reactance to series resistance; with capacitive reactance, $Q = 1/\omega CR$.

When considering a resonant circuit it is often useful to know its equivalent parallel resistance, or dynamic resistance. Denoting it by R_d, and the reactance (inductive or capacitive) by X, the ratio R_d/X is the same as X/R, which is what we know as Q. So if we know that the reactance of a tuning coil in the anode circuit of a valve is, say 1000 Ω, and its Q is 100, then it acts as a coupling resistance of 100,000 Ω. (Because $R_d = QX$). And of course its series r.f. resistance is 10 Ω (= X/Q).

Nowadays most of the interest is focused on those frequencies which the Editor conveniently gathers together under the single abbreviation "e.h.f." (i.e., everything over 30 Mc/s). At such frequencies the concept of a circuit composed of lumped L and C more or less breaks down. That being so, the concept of Q, if it can be made to apply, is more useful than ever, because of the difficulty of measuring L and C and of knowing what they signify when one has measured them. So Q has recently been redefined in more general terms as:

$$\frac{2\pi \text{ times the energy stored}}{\text{energy dissipated}}$$

in the circuit per half-cycle.

Simple lumped circuits such as Fig. 1 are particular cases, in which Q as defined in this general way simplifies to $\omega L/R$ or whatever is appropriate. So accepting the newer definition doesn't make it necessary to unlearn the old. There are, however, a few bogus definitions, such as the reciprocal of the power factor, that ought to be scrapped, however nearly right they may be in most cases.

You may ask how the energy stored per cycle in an e.h.f. circuit can be measured. Well, the most convenient form for definition is not necessarily the most convenient form for measurement; and in this case measurement is best tackled indirectly. It is sometimes possible to measure the decrement, or rate of dying-away of oscillations. But the most generally convenient is the Fig. 7 method, which holds good even with resonant cavities for centimetre waves. Frequency is the most accurately - measurable quantity there is; so the only other thing to provide is an indicator to show when the voltage or current amplitude is 70% of maximum—roughly 3 db down.

Summing up the main points:

(1) The modern definition of Q, completely general in its application, is based on the ratio of energy stored to energy dissipated in the circuit.

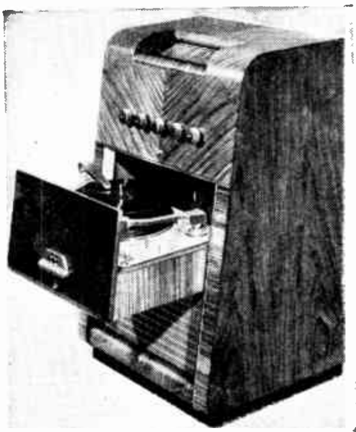
(2) Applied to lumped circuits, this is equal to the ratio of the reactance (purely inductive *or* capacitive) to the series resistance (in its widest sense, covering all losses).

(3) This X/R ratio is also equal to the ratio of V, the voltage across the whole reactance of one kind in a circuit at series resonance, to v, the voltage injected in series—the ratio known as circuit magnification factor (m). But when, as is usual, resonance is judged by the maximum parallel voltage, there is a discrepancy between m and Q, which is negligible unless Q is in the lower single-figure range.

(4) If Q or m is measured by the type of circuit shown in Fig. 3, (such as the usual type of Q-meter, Fig. 6), the result is the apparent Q, or Q′, equal to $Q\left(\frac{C}{C + C_0}\right)$. Since this is almost the only practical way of directly measuring m, in practice m is the same as Q′ (neglecting the discrepancy mentioned above).

(5) Q, however, can be measured by other methods (such as the frequency-variation method, Fig. 7) which give true values directly, and these correspond with the conditions under which tuned circuits are most commonly used (Fig. 5).

NEW RADIO-GRAMOPHONE



A two-position tone control gives normal and extended frequency range on gramophone records in the latest Marconiphone Model ARG19A. The auto-changer handles up to ten 10in or 12in records. On the radio side, a four-valve plus rectifier superhet. covers short, medium and long waves. Three extra positions on the waverange switch give two preset stations on medium and one on long waves. The price is £84 11s. 4d. including purchase tax.