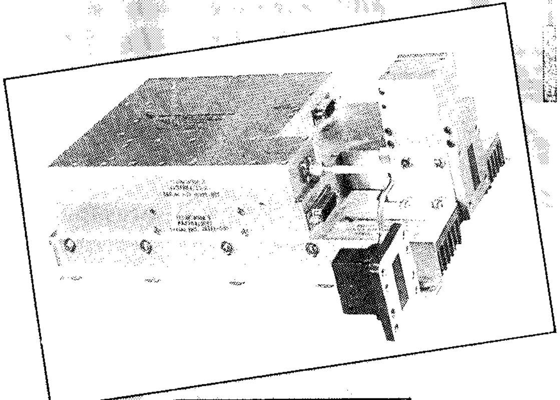
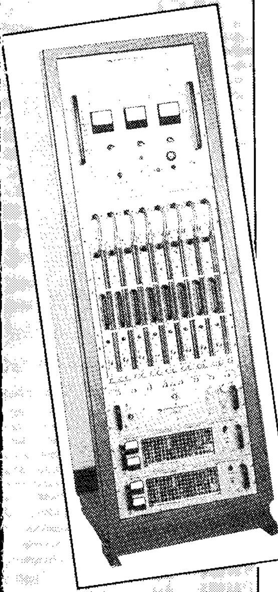
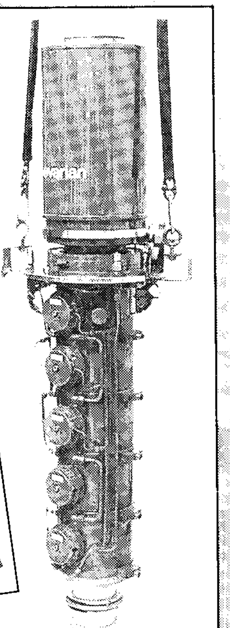
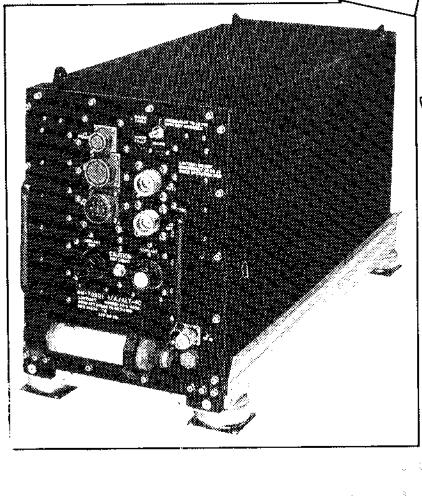
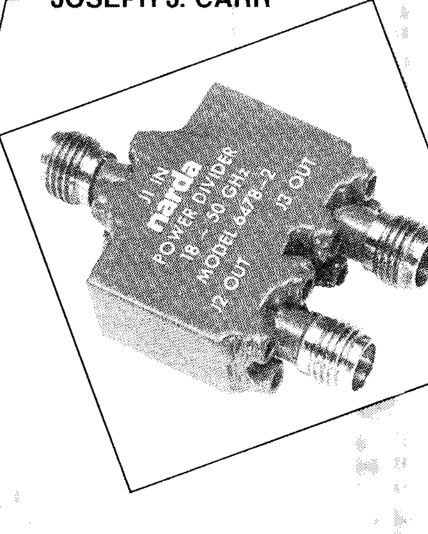


INTRODUCTION TO MICROWAVE TECHNOLOGY









Learn the principles of magnetron tubes.

JOSEPH J. CARR

Last time, we covered the major factors that kept classical vacuum tubes from generating microwaves in normal use: interelectrode capacitance, lead inductance, gain-bandwidth product, and transit-time effects. The major effort in the 1930's and 1940's was to either circumvent those limitations, or exploit them to good advantage.

In a magnetron, electrons accelerate toward the anode as in a classical vacuum tube, but are deliberately returned to the cathode by a magnetic field as their speed increases. If an RF wave of the right frequency is in the cavity when the magnetic field returns the electrons, and the electrons move at the right speed (the phase velocity) they transfer kinetic energy to the RF wave by resonance.

The electrons re-bombard the cathode, and the process repeats. Only those electrons transferring enough energy to the RF wave at resonance reach the anode, and only if they're close enough to be attracted to it. M-type magnetrons are efficient, so very few electrons reach the anode.

The whole point of classical vacuum tubes *is* anode current. The goal of a magnetron is to *not* generate anode current, since an electron reaching the anode can't be reexcited to generate more kinetic energy to transfer to the RF wave by resonance. An RF probe in the cathode-anode interaction space conducts the RF energy to an outside waveguide. The RF wave is the output, not the anode current.

Electric and magnetic fields

There are really four quantities relevant in electromagnetics: electric field intensity E_o, magnetic field intensity H_o, electric flux density D_o, and magnetic flux density B_o. The British physicist James Clerk Maxwell, in the 19th century, found the complex equations governing all four quantities.

The units of electric field intensity are volts per meter, or volts/m. The units of magnetic field intensity are amps per meter, or amps/m. The units of electric flux density are coulombs per square meter, or coulomb/m^2, where

$$1 \text{ coulomb} = 6.25 \times 10^{18} \text{ electrons}.$$

The unit of magnetic flux density is the tesla, where

$$1 \text{ tesla} = 1 \text{ weber/m}^2.$$

The weber is the metric unit of magnetic flux (not flux density), and is the magnetic analog of the coulomb. Just as

$$1 \text{ coulomb} = 1 \text{ amp} \times \text{s}$$

in the electric case,

$$1 \text{ weber} = 1 \text{ volt} \times \text{s}$$

in the magnetic case.

In most simple materials and vacuum, flux density is linearly proportional to field intensity. For the electric case,

$$D_0 = \epsilon \times E_0,$$

and for the magnetic case,

$$B_0 = \mu \times H_0.$$

The permittivity of free space is ϵ_0, and the permeability of free space is μ_0. In other than free space (or vacuum),

$$\epsilon = \epsilon_r \times \epsilon_0,$$

and,

$$\mu = \mu_r \times \mu_0.$$

The permittivity of free space is

$$\epsilon_0 = 10^{-9}/(36 \times \pi) \text{ farads/m} = 8.85 \text{ pF/m},$$

and the permeability of free space is

$$\mu_0 = 4 \times \pi \times 10^{-7} \text{ henry/m} = 1.26 \ \mu\text{H/m}.$$

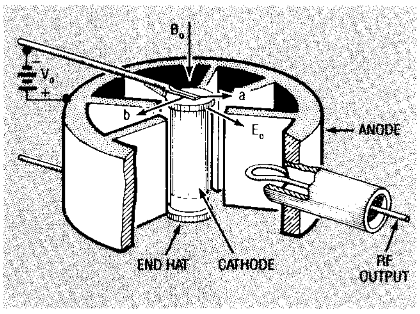


FIG. 1—A BASIC CONVENTIONAL cylindrical or conventional magnetron. The individual chambers between the spokes are the reentrant cavities, and connected to the gaps between the walls of the cavities. They're cavity resonators, metallic enclosures that confine RF energy.

The M-type magnetron

A conventional cylindrical magnetron is shown in Fig. 1. The chambers are reentrant cavity resonators—metallic enclosures confining RF energy, connected at the gaps between their walls. They have an infinite number of oscillating modes, the lowest-frequency one being dominant. At resonance, a standing wave is generated, and the peak electric and magnetic field energies are equal.

There's a DC operating potential V_0 between cathode and anode, while magnetic flux density is in the +z-direction, into the page. The anode is grounded, and the cathode is highly negative. If the anode were positive, both it and the output waveguide would need insulation, and be dangerous to service.

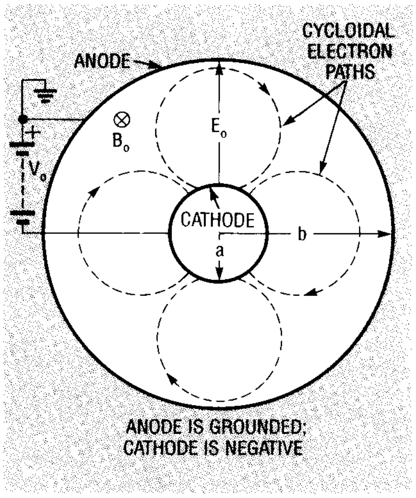


FIG. 2—ELECTRON PATHS IN A conventional magnetron. If an electron is injected into the electric field E_0 between cathode and anode, it accelerates linearly. If the lines of magnetic flux density B_0 is present into the page, and both E_0 and B_0 are properly adjusted, the electron moves cycloidally.

Figure 2 shows electron paths in a conventional magnetron; when electrons are injected into the electric field between cathode and anode, they're linearly accelerated. If the magnetic field is into the page, and both the electric and magnetic fields are properly adjusted, the electron moves cycloidally.

The Hull cutoff condition

Figure 3 shows three views of a simple conventional magnetron. The electric field lines go from anode to cathode, and the magnetic field lines are into the page. For a constant magnetic field, operation is governed by the Hull magnetic cutoff criterion. The first form gives the Hull cutoff potential in terms of magnetic flux density. The second form gives the Hull cutoff magnetic flux density in terms of operating potential. The first form is

$$V_{oc} = \left(\frac{e}{8m}\right) B_0^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2.$$

The second form is

$$B_{oc} = \frac{\left(8V_0 \frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$$

In the above expressions

V_0 is the operating potential,

V_{oc} is the Hull cutoff potential in volts,

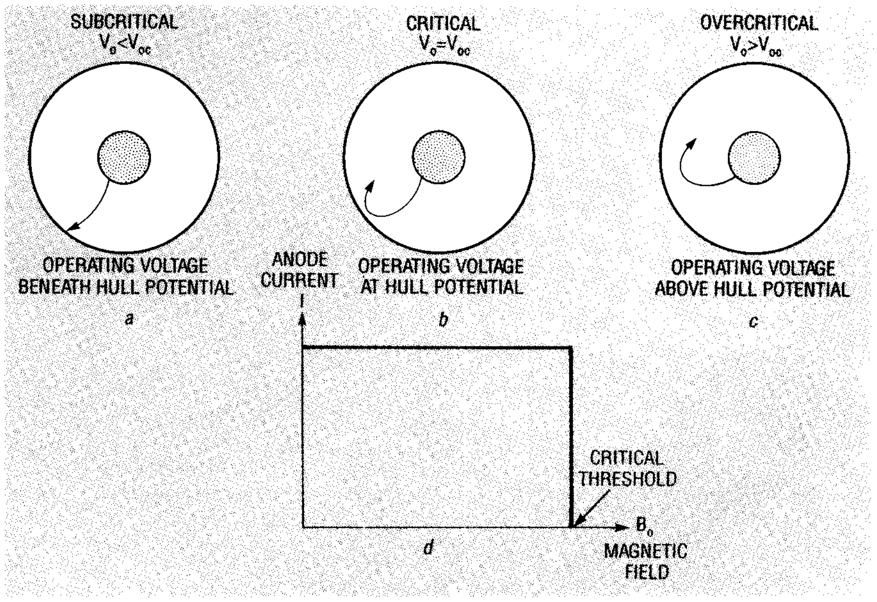


FIG. 3—THE THREE ELECTRON TRAJECTORIES in conventional magnetrons. The lines of electric field E_0 goes from anode to cathode, while those of magnetic flux density B_0 are into the page. The subcritical case (*a*) yields true anode current. In the critical case (*b*), electrons graze the anode, and in the supercritical (cutoff) case (*c*), they never reach it. A plot of anode current I as a function of B_0. is shown in (*d*).

a is the cathode radius in meters,
b is the anode radius in meters,
$e = 1.6 \times 10^{-19}$ coulombs, the electron charge,
$m = 9.11 \times 10^{-31}$ kilograms, the electron mass,
B_o is the magnetic flux density in webers per square meter (Wb/m^2),
B_{oc} is the Hull cutoff magnetic flux density in webers per square meter (Wb/m^2).

The relative values of operating potential and magnetic flux density govern magnetron operation. For the first form, if magnetic flux density is above cutoff for a given operating potential, the electrons don't reach the anode. The reverse holds true for the second form; if the operating voltage is under the Hull cutoff potential for a given magnetic flux density, the electrons again don't reach the anode.

Figure 4 shows the three conditions for the second form of the Hull cutoff criterion. Figure 3-*a* shows the subcritical case, where the operating potential is below cutoff, with a true anode current. Figure 3-*b* shows the critical case, where the operating potential is at cutoff, and the electrons graze the anode before returning. Figure 3-*c* shows the supercritical case, where the operating potential is above cutoff, and the electrons are deflected back before hitting the anode. Figure 3-*d* shows anode current I as a function of magnetic flux density B_o.

Magnetron anodes

The circular anodes in Fig. 3 have an infinite number of modes, and are useless. Real magnetrons use cavity resonators like those in Fig. 1; each is a resonant L-C tank as in Fig. 4; the cavity walls fix inductance, the wall conductivity fixes resistance (not shown). Several versions are shown in Fig. 5; each is a single metal block, and B_o appears in Fig. 5-*d*.

Consider the hole-and-slot block in Fig. 5-*a*; the others are similar. For unstrapped (unshorted) anodes, the tanks are in series, as in Fig. 6-*a*. However, if alternate cavities are strapped (shorted) as in Fig. 6-*b*, the anode is an array of parallel L-C tanks, as in Fig. 6-*c*. That's normal for most magnetrons; due to the strapping of alternate cavities, adjacent resonators are 180° out of phase.

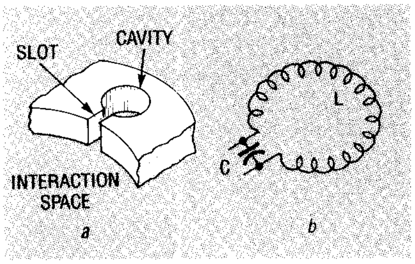
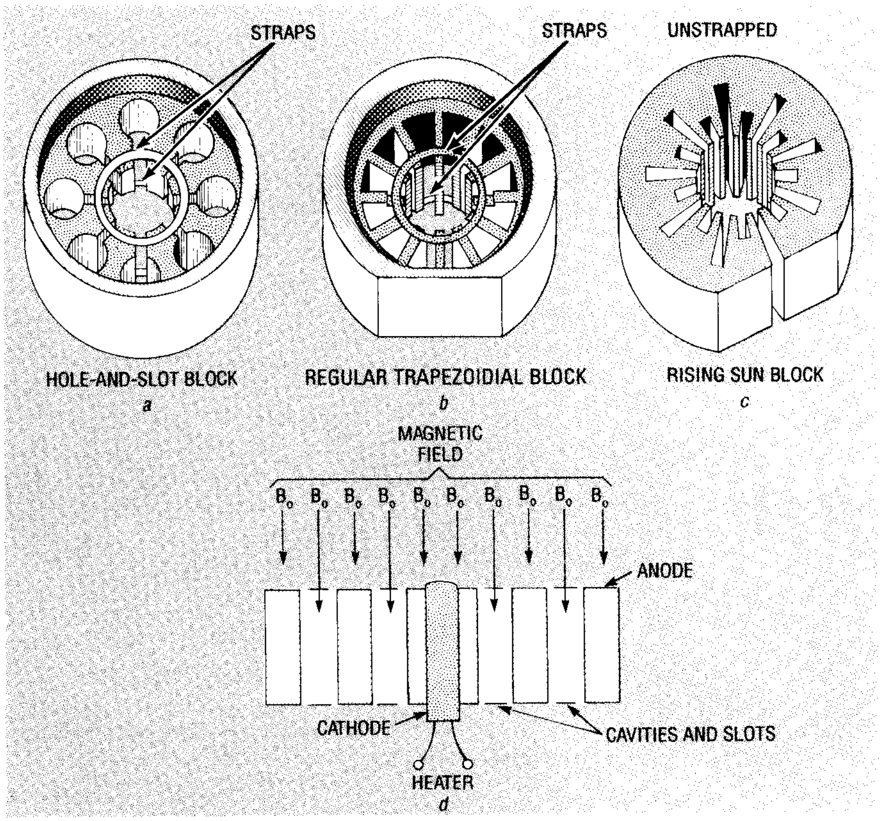


FIG. 4—THE CAVITY RESONATORS IN Fig. 2 act like a resonant L-C tank. The cavity walls primarily contribute to cavity inductance, while the spacing between the walls of the gap at the base of the cavity primarily contributes to cavity capacitance.



HOLE-AND-SLOT BLOCK
a

REGULAR TRAPEZOIDAL BLOCK
b

RISING SUN BLOCK
c

d

FIG. 5—SEVERAL DIFFERENT MAGNETRON ANODES; a hole-and-slot block (*a*), a regular trapezoidal block (*b*), and a rising sun block (*c*). In each, the cavities and anode are made from a single metal block; B_0 is arranged as in (*d*).

Pi-mode operation

Most magnetrons work in π-mode, where the phase shift between adjacent resonators is π radians, or 180°. The radian is an alternate unit of angle, where 1 radian = 57.3°. When a magnetron is turned on, the electron cloud shock-excites (rings) the cavities, setting up a spatially- and time-varying electric field in the cathode-anode interaction space, adding to that due to the operating potential. It varies the acceleration of the electrons due to the operating potential V_o, modulating their velocity and V_o density, creating a "bunched up" pattern.

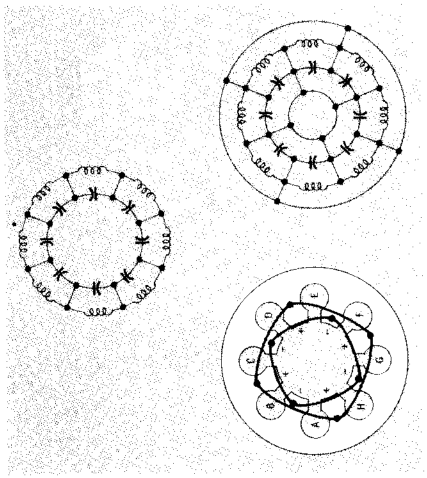


FIG. 6—EACH HOLE AND SLOT CAVITY IS A parallel-resonant L-C tank. In unstrapped (unshorted) anode, the tanks are in series, as in (*a*). If alternate cavities are strapped (shorted) as in (*b*), the magnetron becomes an array of parallel L-C tanks, as in (*c*). Magnetrons are normally strapped, so adjacent resonators are 180° out of phase.

Figure 7-*a* shows the electric field due to the operating potential for a hole-and-slot block, and

for a trapezoidal block in Fig. 7-*b*, both in π-mode. For oscillation to occur, the electron velocity must equal the phase velocity of the RF wave, so resonance can transfer kinetic energy into RF energy. If that occurs, the electron cloud keeps ringing the cavities, generating RF waves. The electron velocity is the ratio of the electric field from the operating potential to the magnetic flux density, or

$$v_o = E_o/B_o.$$

At that speed, the electrons lose energy to the RF wave, slow down, and return. During oscillation, there's no anode current, as shown in Fig. 3-*d*. Since the magnetic field is perpendicular to the electron motion, the centripetal acceleration inward from the magnetic field equals that radially outward from the electric field.

A reentrant cavity resonator is a "slow-wave" structure; its purpose is to slow down the phase velocity of an RF wave (more below), so it interacts with an electron beam. The cavity is designed so oscillations occur only if the total phase shift around the anode is a multiple of 360° or $2 \times \pi$ radians, creating a standing wave. For an N-cavity anode, the phase shift between adjacent resonators is

$$\phi_n = (2 \times \pi \times n)/N,$$

where n is the mode of oscillation.

All RF waves have a group velocity and a phase velocity. Group velocity v_G is the speed of energy propagation, equal to the speed of light in free space divided by the refractive index of the medium an RF wave passes through. Phase velocity v_P is the speed of phase propagation, the rate phase varies with distance, the speed of light multiplied by refractive index of the medium. The refractive index of a medium is always the square root of the product of its relative permeability and permittivity, so that

$$v_G = c/\sqrt{\mu_r \times \epsilon_r},$$

and

$$v_P = c \times \sqrt{\mu_r \times \epsilon_r},$$

where

$$c = 3 \times 10^8 \text{ m/s}$$

is the speed of light in free space. In the vacuum of a magnetron, both group and phase velocities equal c.

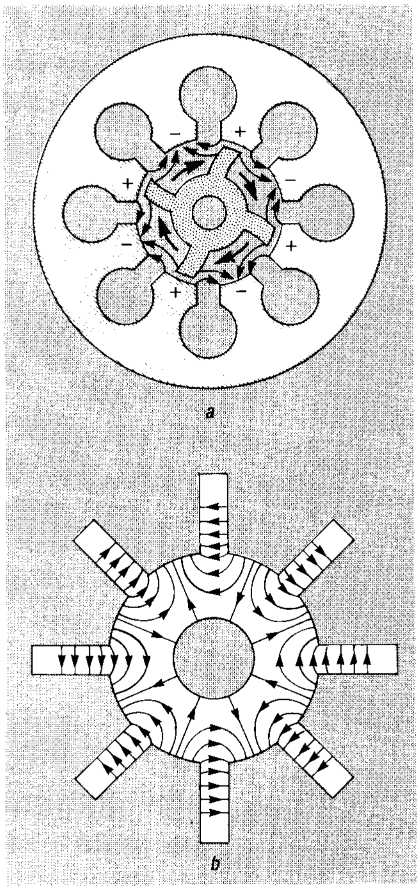


FIG. 7—THE SHOCK-EXCITED, spatially-varying π-mode E_o for a hole-and-slot block in (*a*), and a trapezoidal block in (*b*), both with N = 8. They cause nonuniform electron acceleration resulting in velocity and density modulation, and "bunching up" of electrons.

Another way to think about these two speeds is that group velocity means, "So many joules of energy per square meter pass a point per unit time," while phase velocity means, "So many radians of a wave pass a point per unit time." For a magnetron to oscillate, the electron tangential speed must equal the phase velocity, or

$$v_o = v_P.$$

In Fig. 7, the electric fields shown are for two different eight-cavity blocks, or N = 8. For those two blocks, the mode can be found since

$$\phi_N = (2 \times \pi \times n)/8 = \pi,$$

so that those two anode blocks exhibit fourth-order modes, or n = 4. The successive adjacent cavities create a traveling RF wave along the surface of the slow-wave structure. The "spokes" in Fig. 7-*b* represent segments of the space-charge cloud as it revolves.

For a magnetron operating in π-mode, the number of cavities has to be even, or N = 2, 4, 6,.... Since the phase shift between adjacent cavities is 180°, each pair of cavities shifts the phase 360°, or one cycle, maintaining a standing wave. The operating frequency of a magnetron is the speed of light divided by the product of the number of cavities N and the mean distance L between them, or

$$f = c/\lambda = c/(N \times L).$$

The Hartree cutoff condition

The Hull condition expresses the cutoff potential in terms of magnetic flux density, or vice-versa. A more complex companion condition relating magnetic flux density to the cathode-anode spacing and the operating potential is the Hartree criterion. For the conventional magnetron

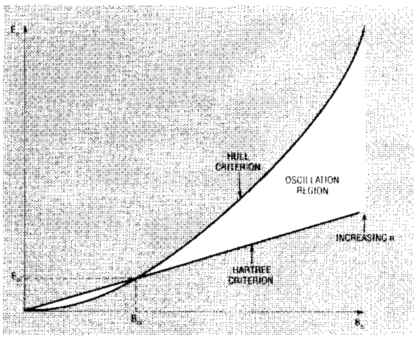


FIG. 8—THE RELATIVE EFFECTS of the hull and Hartree criteria, plotting the electric field E_o versus the magnetic flux density B_o. The electric field is directly proportional to the operating potential V_o, since the cathode-anode spacing is fixed. The Hull criterion is above, the Hartree criterion below, and they intersect both at the origin and when $V_{oc} = V_{oh}$. If $V_o > V_{oi}$ and $B_o > B_{oi}$, oscillation occurs; if not, the RF energy is cutoff.

$$V_{oh} = \frac{2\pi f B_o}{Nc}(b^2 - a^2).$$

In this expression

V_{oh} is the Hartree potential in volts,

B_o is the magnetic flux density in webers per square meter (Wb/m^2),

f is the frequency in Hz,

N is the number of resonators,

b is the anode radius in meters,

continued on page 76