ome thoughts on transformers

What sets the limits in design

by Thomas Roddam

If you want a transformer you may set about getting it in any one of a variety of ways. At first sight the easiest is to work for a large organization which has its own group of transformer designers. You simply say what you want, wait, and up comes something which is too big for the job. That group was not hired to make your life easy, but to keep the number of sizes of lamination and bobbin held in the stores to a minimum. The experts are not usually too good if you want something really subtle, either. The extreme in elaboration that I can think of was the designer who had to design a new grinding machine to get the close tolerance he needed for the bobbin of a really closely defined transformer. Other ways are to use the nearest item you can find in someone's stock list, wind it yourself on the kitchen table, or find one of the smaller manufacturers who will make a single transformer, either because one is all you want, or as a prototype.

This last solution splits, in theory, into two possibilities. Either you go cap in hand, and say what you want the transformer to do, leaving it to the manufacturer to design it when he has time, or you design it yourself. If you leave it to the manufacturer he may have to fit it in with the main task of keeping the business running, or he may have a mysterious "designer", who is never seen and who, I suspect, either does this in spare time from his proper job, or is the lab boy at the local tech. There is a third possibility, the one which set me thinking about this article when I first heard about it: you bodge up a design. You take an existing design and ask for something the same, but different. The specific example which first introduced me to this method was the man who had been buying transformers for a 10 watt amplifier system. He changed nothing but the number of turns and then complained that he could not get 20 watts out with a higher supply voltage.

It appears to me that even if what you require is extraordinarily simple, for example a unity ratio isolating transformer, you should do some of the design work yourself. That simple isolating transformer may land you in trouble. I have met off-the-shelf units which, in the interests of economy, were designed to work at rather high flux densities. In consequence there was a sharp current peak which led to a good deal of confusion. The whole situation has become more complicated with the need to build power transformers to work at higher frequencies. If you want to handle 100 watts at 1kHz, or 20kHz, you will not get much help from your little man round the corner. You will not get much help from most of the textbooks, either.

What makes the variety of transformers interesting is the fact that the rules seem to change. Of course the essential theory is the same, but the limiting factor for one set of conditions turns out to be unimportant for another set, because a factor which looked after itself has become predominant. It is this question of what sets the limits which I propose to examine.

The simplest transformer we use is the ordinary 50Hz mains transformer. The two limits in normal design are the flux density in the core, and the current density in the wire. Magnetizing current, as such, is not often a problem; nor, to my mind, is core loss. It is wrapped up by the matter of flux density, which always needs to include an idiot factor. If you provide taps, will someone set to 220V and connect to a nominal 240V which is actually 250V? Will the transformer be used in one of those places where they would rather have some power at 45Hz than a black-out at 50Hz? Current density is quite simply a matter of the transformer getting hot. We should consider the regulation, or so the books say, but we are more and more passing the job of controlling the final level to some clever circuits, and in many applications we find that we should like even more transformer resistance than we dare include.

A very simple guide to mains transformer design which I found somewhere or another, and which seems to give a good place to start, is that the core cross-sectional area should be

$$W^{\frac{1}{2}}/5\text{in}^2$$

where W is the power to be handled. I know we should not use inches, but the cores people's stock is all described in inches. A 25W transformer should, on this basis, fit nicely on a square stock with a one inch centre limb. I shall have a go at deriving this expression in an appendix, but I have a nasty feeling that with my choice of parameters I shall get a different numerical factor. The object of all these guide equations is really nothing more than offering a good starting point for the first rough design. A better method is to look back at earlier designs, if you have any, or to try to work out from the catalogue what the other chap did, at least as far as core size is concerned.

The ordinary, everyday, aspects of the design you must look up in the book. Now we are all using silicon rectifiers straight into capacitance smoothing the addition of a screen is even more important. You may want to know the magnetizing current, for calculating the protection circuit, but it really is safer and easier to measure it.

When we leave the simple world of the 50Hz power transformer it seems natural to move to the 400Hz power transformer. If we were to do nothing special, but just design as before for a reasonable flux density just below saturation, and take no further thought, we should be in trouble. The laminations which were gently warm would now be very hot indeed. Each lamination is, of course, of finite thickness, which for the bread and butter world is 0.015in. The thickness is a small short-circuited turn, and there are rather a lot of them. Each of these turns is rather loosely coupled to the primary, and the effect of the short-circuited turns depends on both the coupling and the resistance of the turn. A detailed analysis was done by Caver, but it is pretty obvious that if we use thinner laminations the coupling to each one will be weaker, and its resistance higher. The iron-masters have decided for us that 0.004in is the right thickness to use for a 400Hz: there is no point in doing a lot of calculation and finding that it should be 0.003 or 0.005. The chaps who make the stuff think that Milton was writing about them.

A difficulty with thin laminations is that they are so thin. Fortunately we can get C-cores, which are easy to put together, have rather better magnetic properties and, because so many users prefer them, have made it almost impossible to find a source for small quantities of the 0.004in laminations. You do not need a guidance equation for C-cores: the maker tells you the power he, or his predecessor, would expect each size to handle.

Apart from this matter of using the thinner material, the key criteria are the same at 400Hz as they were at 50Hz: flux density safely below saturation, current density below overheating.

It is interesting to notice that we could have made our 400Hz transformer with the 0.015in laminations if we had kept the flux density very low. Of course this would have meant using a much bigger transformer. But this is exactly what we do when we construct an audio output transformer. At the largest signal level at the lowest working frequency we allow the flux density to be moderately high. Suppose we choose $B = 10000\text{G}$ at 40Hz. For the same signal level at 400Hz the flux density will be only 1000G. Observations on real transformers show that the eddy current loss effect is not significant. If we use 0.004in laminations to make transformers to operate from about

All this discussion has been in terms of a square stack of no-waste shape. It is fairly clear, I think, that if we vary the thickness of the stack we shall vary the voltage which can be applied to the winding for the chosen flux density. This assumes that we keep the same number of turns of the same wire gauge. The transformer wattage is thus directly proportional to the stack width. If we go into more detail we shall find a limiting process produced by the increasing turn length, but the mechanical difficulties are usually the dominating ones. When we turn away from the no-waste lamination we can reason roughly like this: keeping the turns the same for a given centre limb area, the current will be proportional to the window area. Thus the wattage is proportional to the window area.

Some of the results do not agree with the results of a perfectly general analysis. It is unfortunate that most analytical solutions to problems explain why such and such does so and so. We do not want to know why this transformer gets hot at a loading of 150 watts: we want, with less scientific precision, a transformer that stays cool, and is manufactured from standard parts. General solutions are always attractive when you are doing the theory, because you wrap up the whole problem in one bumper bundle: the bundle is an end in itself.

I had intended to conclude with the corresponding expression for inductors carrying direct current: indeed, I have done so in the appendix. The result is to give a core area of

$$A = (VI_2)^{2/3}/25\text{in}^2$$

At first I was rather unhappy about the result which showed up, which did not take account of the range of working currents. This result looks quite sensible, and a quick check on a 100-watt unit, say 100V, 1A, shows the transformer to have a core area of 1.5in^2 and the inductor to be 0.85, or just over half the size. Notice that, like the statisticians who draw little men, or little ingots of gold, to compare different systems, I have not been too clear about what size means.

Any design is a compromise: if you can save energy in getting your rough solution you can use the time to get the best compromise.

Appendix

Core properties based on one no-waste lamination

The no-waste condition ties all the lamination dimensions together, so that a standard shape can be used to establish guide formulae. The figure shows how a pair of Is is stamped out of each pair of Es. The window must have dimensions a by $3a$ for this simple picture to be true. A further simplification for the analysis is to assume that we make the core thickness $2a$, giving a square stack. The coil winders find this very attractive.

The core area is then $4a^2$.

The window area is $3a^2$.

The mean magnetic path is $12a$, if we consider what happens if we slit the E down its centre line.

The volume is $48a^3$.

In spite of the fact that all the bright young men will complain, the basic dimension a is expressed in inches, because that is how the cores are specified.

The volts/turn for this core is given by

$$\frac{V}{N} = \frac{4.4BA_f}{10^8} = 4.4B.4a^2.6.45f.10^{-8}$$

$$= 113.5a^2Bf.10^{-8}$$

The window area is not full of copper. The assumption is that one half is primary and one half secondary, that copper occupies $\pi/4$ of the available space and that only a fraction p is left after we have provided a bobbin and all the other wastage. The primary copper thus occupies an area of

$$\frac{\pi}{4}.\frac{1}{2}.p.3a^2 = \frac{3\pi}{8}p.a^2$$

If we make

$$p = 0.85 \text{ and operate at } 1000\text{A/in}^2$$

$$\text{or} \quad p = 0.565 \qquad 1500\text{A/in}^2$$

we get the very agreeable result that

$$NI = 1000a^2$$

Multiplying this by the expression for V/N:

$$VI = 113.5Bfa^4.10^{-5}$$

If now $\quad B = 12.35 \times 10^3$

$$VI = 14fa^4$$

And at 50Hz

$$VI = 700a^4$$

The core area was, as we saw

$$A = 4a^2$$

so that $\quad VI = \frac{700}{16}.A^2 = 43.8A^2$

Now VI is the power which the transformer will handle, and to find the size of transformer for a given power, $W = VI$, we simply take a core area of

$$A = (W)^{\frac{1}{2}}/6.6$$

The difference between this and the form $(W)^{\frac{1}{2}}/5$ which I have been using on unknown authority, can be attributed to a number of factors. The unknown x may not have used no-waste laminations and he certainly used different values for the flux and current densities. If we allow for the frequency to be 20% low, we should get a figure of 6, but that seems to be over cautious.

Of course it does not matter. It is extremely rare to know the exact power which a transformer will need to handle. This is an expression for guidance, and should not be regarded as anything more.

At 400Hz the situation is, as I have pointed out, rather different. We are given the ratings for C-cores, which are not the no-waste shape anyway. What is also significant is that the flux density can be higher. Forgetting all this, and just putting in 400 for f

$$VI = \frac{5600}{16}A^2 = 350A^2$$

$$A = (W)^{\frac{1}{2}}/18.7$$

The weight of the core will be about $12a^3$lb, and if we take what I think is a rather low core loss figure of 1W/lb at 50Hz the core loss will also be $12a^3$. The area of core surface which is not shielded by the bobbin is $72a^2$, so the dissipation of heat must be

$$12a^3(W)/72a^2\,(\text{in}^2) = \frac{a}{6}W/\text{in}^2$$

For values of a less than about 2in, which is the size we are always considering, this implies quite a moderate temperature rise.

Let us now turn our minds to the magnetizing current. The inductance of the primary is given by

$$L = \frac{1.259N^2 4a^2.6.45\mu 10^{-7}}{12a.2.54}$$

$$\simeq N^2 a\mu 10^{-6}$$

The magnetizing current is

$$I_m = V/2\pi fL$$

and $\quad V = (4.4BN.4a^2.6.45f)/10^8$

giving $\quad I_m = \dfrac{113.5BNa^2f}{2\pi N^2 a\mu f.10^8.10^{-6}}$

$$= \frac{18Ba}{N\mu 100} = \frac{0.18Ba}{N\mu}$$

The useful current, the one we use for working out the power, is

$$I = 1000a^2/N$$

so that

$$\frac{I_m}{I} = \frac{0.18B}{1000\mu.a}$$

or, to make it a bit simpler, we can approximate to

$$I_m/I = B/5000\mu a$$

For the input inductor of a 50Hz full-wave rectifier system we already have one simple rule:

$$\text{Inductance } L = (V/I_1) \times 10^{-3}$$

to maintain continuous current flow. Here V is the output voltage and I_1 the minimum working current. A designer will be lucky if he can get an energy storage density given by

$$\frac{LI_2{}^2}{\text{Vol}} = 0.1$$

where I_2 is the maximum current, or

$$LI_2{}^2 \simeq 50a^3 \times 0.1 = 5a^3$$

This is, of course, only one point on the Hanna curve. In accordance with the rule that numbers are chosen to give simple answers, let us take

$$I_2 = 5I_1$$

Then $\quad LI_2 = 5V.10^{-3}$

$$LI_2{}^2 = 5a^3 = 5VI_2.10^{-3}$$

$$a^3 = (VI_2).10^{-3}$$

$$a = (VI_2)^{1/3}/10$$

so that the area of the centre limb is

$$A = 4a^2 = (VI_2)^{2/3}/25$$

1kHz upwards we can see the effect of the eddy current loss. Instead of the frequency response being that of an *LR* circuit it becomes deformed. Not much, it is true, but the effect is observable.

Power applications of higher frequencies have been with us for much longer than most people think but with the development of the transistor and the thyristor it became so much easier to get powers in the range from tens of watts to tens of kilowatts that the attitude of the power user became completely transformed. One range of frequencies in common use is roughly 1kHz to 1.5kHz. I do not wish to go into matters of circuit design, but there are often good reasons when the older practice of using a tuned transformer is not practicable. The transformer designer is required to produce, let us say, a transformer to handle 200VA at 1kHz, with the primary and secondary volts specified.

In one sense there is no special problem. A probable core is selected, and the number of turns needed to give the right flux density is examined to see if they can be wound with wire which will carry the current. Then, just as we used thinner core material when we changed from 50Hz to 400Hz, so we must seek out the appropriate thickness for 1kHz. Unfortunately this drives us into the country of "specials", the things you can't get, and couldn't afford if you could get them. If you just use 0.004in material at its full flux density the core will get very hot, which is particularly undesirable when all the power being wasted has been produced rather expensively with semiconductor devices.

It is at this point that we fix a new design criterion, or perhaps more correctly a new starting point. We choose our core loss. The procedure is one of ruthless guesswork. Guess the size of core which will be needed: this gives us the weight. Guess a reasonable core loss, perhaps 3% of the total power. From these two figures we can find the core loss per unit weight and then turn to the manufacturer's data sheets to find the approximate flux density. From now on the design is straightforward but, at first, tedious. If your guess is wrong, and the transformer is obviously too big or too small, you must guess again. If the first shot was not too far out, the second design will be satisfactory. The beginner may need to have a third shot, and the more advanced designer, once the size is about right, may want to vary it to trade iron losses against copper losses. A point worth noticing in this kind of transformer is that iron losses are always with us, even if we are not using any output. This can be significant in battery operated systems which are only lightly loaded for most of the time.

For operation at high audio frequencies, that is above the classic 400Hz, it is tempting to consider the use of nickel-iron alloys. These are available as thin laminations, in a range of sizes, and in materials of high permeability and high resistivity. In an ideal world they would be perfectly suited for many applications. For some reason which I cannot understand, obtaining any of these laminations is an extremely frustrating operation.

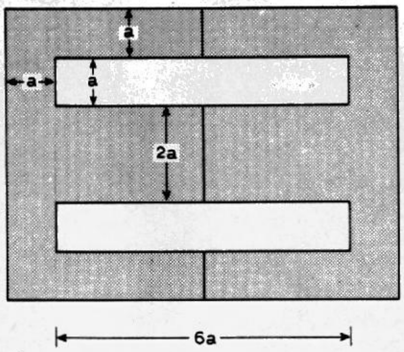

Fig. 1. Genesis of the no-waste lamination.

The really fashionable power transformers nowadays are those used in transformerless power supply units. It will not surprise the older readers who remember the domestic comments about wireless to learn that one design, at least, of these transformerless supply units has three transformers inside it, instead of the usual single transformer. As every schoolboy knows, the only phrase written by the great Macaulay which remains in my memory, these power supplies simply rectify the mains, to give some 300 odd volts, and then use an inverter running at some 20–50kHz to get some transformable a.c. The part of the system where you have to be clever, or extra clever, is passing the message back from the output to the inverter side, where all the control takes place and which is quite firmly connected to the mains. When you recall that you can get these units which provide 100A at 5V you will see that the control must be on the primary side, where if the efficiency were ideal the current would be less than 2A.

In fact these are only the latest in a long line of d.c. to d.c. converters, and are related to other power converters. It is a new high-speed, high-current rectifier which has brought the possibility of this particular system into being. The lower power systems, and the 10–20W level has had a good many applications, have been very tempting subjects for operation in the 20–50kHz range, but there are some rather interesting problems in the design of the transformer. At first sight it is attractive to use a toroidal core of the very thin nickel iron material which is, in theory, available. The thinness is essential to avoid eddy-current losses. Toroids are, however, a nuisance for winding unless you have a suitable winding machine, and even then there are some problems. Another serious difficulty for most of us is the problem which you meet when you learn to ride a bicycle: it is the problem of getting started. To get one core is much more difficult than getting 100.

The answer, if you have a need for only one unit, or as happens if you are selling to the impoverished Third World, perhaps fifty units, is to use ferrite cores. These are cheap and are easily available. The choice is then between the pot cores and the double E's or E and I forms. Pot cores have the great advantage that they are self-shielding. The external field is very small, and this can be important. However, these cores are basically designed for producing inductors. The important thing, when you are making an inductor, is that you should be able to bang on a fixed number of turns, and come hell or high water you should get a defined inductance. I know that there have been changes since the days when iron filings were stuck to sheets of paper (ferrocast) or little spheres of carbonyl iron were all glued together with something or another but in spite of the wonders of progress the permeability of ferrites is not strictly defined. Inductor cores are therefore made to have fixed permeability by the simple process of introducing an air gap. The apparent permeability is therefore very low.

If we were to construct a high frequency transformer ignoring this factor we should carry out our design calculations in terms of the flux density, and the important detail of getting enough copper. We should take account of the rather tedious detail that ferrites do not get the heat away as well as laminations, and cannot stand a high internal temperature gradient. But after all this, we might still be in trouble. The devices must carry the useful current and the magnetizing current. It is the same problem as the elliptical load line we met so long ago in audio amplifier design.

I am well aware that ferrites do not come in the no-waste proportions, least of all the pot cores. In practice, in order to get low leakage inductance, a ferrite-cored transformer will be under-filled, and anyway, we are after guide-lines. The ratio of magnetizing current to useful current is derived in the appendix, and is

$$\frac{I_m}{I} = \frac{B}{500\mu a}$$

If we take $B = 2000$

$\mu = 100$

we get $\frac{I_m}{I} = \frac{1}{25a}$

Remembering that a is half the centre limb width of an E, and is thus, on a typical core, about ⅕in, we get

$$\frac{I_m}{I} = \frac{1}{5}$$

Things are really worse than this. We are thinking about d.c. converters, which operate with square waves. This value of I_m is the sine-wave r.m.s. current, but the actual current is a linear run up, and the unhappy devices concern themselves with the current peak. The devices must be bigger, or driven harder, and as this current is handled by the devices the losses will be higher. We must, therefore, use a material and core style which gives us the highest possible permeability. The alternative is to increase the size, both to increase a and also to allow us to reduce B.

I am not concerned here with the right answers: the important thing in beginning a design is to ask the right questions. The magnetizing current question is one which we need to ask in any low permeability situation, right back to the old-fashioned output transformer in the anode of a single pentode. The general question of the rough size is worth asking yourself even if the actual work of designing the transformer is to be passed on to someone else.