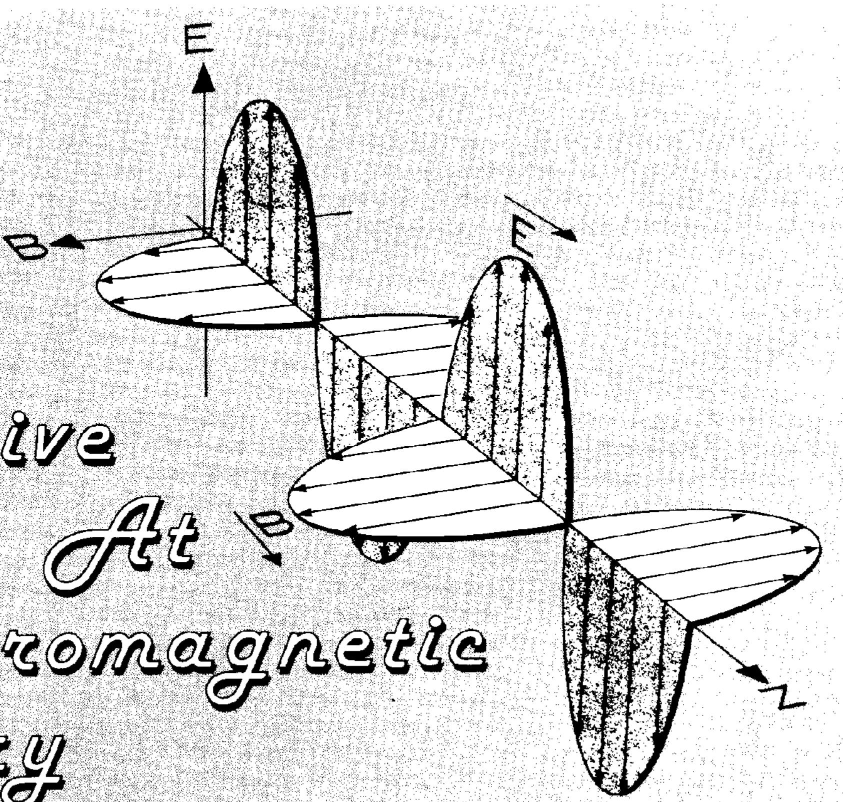
An Intuitive Look At Electromagnetic Theory



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SOME OF OUR READERS MAY NOT BE aware that 1991 marks the 100th anniversary of the third edition of James Clerk Maxwell's *A Treatise on Electricity and Magnetism*, the ultimate reference on electromagnetic theory. What better way to recognize the impact that Maxwell had on the study of electromagnetics than to present the first of a series of articles in **Radio-Electronics** on the subject? In this edition, we will take a physical, intuitive look at the basics of electromagnetism, how they relate to some common electronic components, and how to interpret some of the complex mathematical symbolism. Only a familiarity with vector algebra is needed.

Maxwell's equations were first formulated in 1873. In his first publication, a mathematical foundation for relating electric and magnetic effects were given. In the Preface to the 1891 edition, J.J. Thomson noted that most of his students had difficulty with some aspects of electromagnetic theory. One hundred years later, not much has changed in that regard. One reason is that electromagnetic theory requires knowledge of some involved mathematics such as vector and tensor calculus and integral-differential equations.

Maxwell's idea that a changing electric field gives rise to an associated magnetic field developed from an intuitive sense for the natural order in the world. By presenting physical concepts in such an "intuitive" way, the reader will find it easier to understand Maxwell's equations, and his mathematical approach. Let's begin by examining the concept of an electric field.

The electric field

A scaler can be thought of as a quantity that can be completely characterized by its magnitude. Some examples of scaler quantities are mass, time, and volume. A scaler field is simply an extension of the scaler concept. It is a function of position that is specified by its magnitude at all points in a region of space. Land elevation is a two-dimensional scaler field because at each point of latitude and longitude there is an associated height above sea level. Air temperature is an example of a three-dimensional scaler field. With the appropriate instrument one could measure the height, or temperature, at each point. A scaler quantity is symbolized by a letter, such as h, for height.

A vector is a quantity that is characterized by its magnitude and direction. Some examples of vectors are velocity, acceleration, and force. A vector field is a function of position that is specified by its magnitude and direction at all points in a region of space. An example of a vector field is air velocity, where at each point in space, the magnitude and direction of air flow can be measured with the proper instrument. Vectors are often symbolized by letters with arrows above them, however, we will use boldface letters to indicate vectors.

When using vector notation, **A** is a vector with a specific magnitude and direction, and −**A** is a vector of the same magnitude but pointing in the opposite direction. Vectors are illustrated graphically by arrows, which have a direction and a corresponding length, which is proportional to the magnitude.

The field concept allows us to associate something that happens at one point with what hap pens at another point even though there may be no material objects connecting those points; examples are air temperature and velocity fields. Although we will not be directly concerned with them here, there are other types of fields, such as tensor fields, that assign a set of three vectors to each point in space, or quantum fields that assign mathematical operations to each point in space-time.

Electric charges

Experiments have shown that electric charges are either positive or negative. Like charges repel each other, unlike charges attract. The unit of charge is the Coulomb, C. The smallest magnitude of charge, e, is equal to

$$e = 1.60 \times 10^{-19}\text{C}.$$

Charge follows the principle of conservation, which states that the net sum of all charges in an isolated system remains constant. A charge can be moved, but it cannot be created without the creation of an equal and opposite charge.

Experiments by Coulomb showed that if a charge, q_1, was placed at a point in empty space, nothing appears to happen. But if another charge, q, is placed at some other point, as shown in Fig. 1-*a*, it will experience a force in newtons

$$\mathbf{F}_c = \left[k\frac{qq_1}{r_1^2}\right]\mathbf{r}_1$$

where $\mathbf{r}_1$ represents a vector of magnitude 1 (a unit vector which defines the direction) directed from q_1 to q. $\mathbf{r}_1$ is the separation distance in meters. The constant of proportionality, k, is a number that is chosen to make the units work out. Coulomb's constant, k, has the value in a vacuum of

$$k = 9.0 \times 10^9\ \text{N·m}^2/\text{C}^2$$

The value of k in air is slightly greater. Using the mks system, the constant k can also be written as

$$k = 1/(4\pi\epsilon_0)\ \text{N·m}^2/\text{C}^2$$

which will give familiar units such as volts, ohms, and amperes. ϵ_0 is the permittivity of free space, and is equal to

$$\epsilon_0 = 8.85 \times 10^{-12}$$

The Coulomb force, $\mathbf{F}_c$, on a charge, q, will have a magnitude proportional to the product of the charges, and inversely proportional to the square of the separation distance. That force will also be directed away from q_1. If one of the charges is negative, then the direction will be opposite. That force tends to provide an acceleration, **a**, to q in the same direction. There is, of course, an equal and opposite force on q_1, and Coulomb's law for that is written by simply redefining **r**.

If there is a number, n, of point charges instead of just q_1 present, as shown in Fig. 1-*b*, the force vector of each would all add vectorally to give the total force

$$\mathbf{F}_c = \left[k\frac{qq_1}{r_1^2}\right]\mathbf{r}_1 + \ldots \left[k\frac{qq_n}{r_n^2}\right]\mathbf{r}_n = kq\sum_{k=1}^{n}\left[\frac{1}{r_k^2}\right]\mathbf{r}_k q_k$$

The fact that the vector forces add in this manner is called linear superposition.

If a charge q is spread out over a region of space instead of being located at one point, we consider the charge by dividing it up into an infinite number of infinitesimal charges, dq, and sum the contributions from each. The force that is exerted on a charge q_0 at another point is given by the calculus notation

$$\mathbf{F}_c = kq_0\int(1/r^2)\mathbf{r}\,dq$$

where the integration symbol $\int$ can be "read" as the sum of an infinite number of infinitesimal contributions.

The electric field E

Coulomb's law defines the force only at one point where q is located. It does not define a field in the sense used here, but it provides a starting point to develop the idea of an electric field. Suppose we make q a very small positive charge and use it as an instrument to explore all points other than where q_1 is located. Since q experiences a force $\mathbf{F}_c$ at every point it is placed, we get the impression that the condition of space is affected by the presence of q_1. We can amend the statement that "if q_1 were alone in space, nothing appears to happen" to "if q_1 were alone in space, then space has the propensity to exert a force on another charge, if it is present, according to Coulomb's law." Since that inclination appears to apply to space, independent of any q, we divide q out of Coulomb's law to obtain a definition of the electric field (also called electric field intensity)

$$\mathbf{E} = \frac{\mathbf{F}_c}{q} = \left[k\frac{q_1}{r_1^2}\right]\mathbf{r}_1$$

which can be thought of as a measure of the propensity. $\mathbf{r}_1$ is a unit vector pointing from q_1 to whatever point in space is being considered and r_1 is the distance. That assigns an **E** vector to every point in space (except at q_1 where $r_1 = 0$).

In the case of a number of point charges, n, the **E** field is obtained by linear superposition

$$\mathbf{E} = k\sum_{k=1}^{n}\left[\frac{1}{r_k^2}\right]\mathbf{r}_k q_k$$

For a spread out charge distribution, summing by integration gives

$$\mathbf{E} = k\int\left[\frac{1}{r^2}\right]\mathbf{r}\ dq$$

Figure 2 illustrates the **E** fields for a number of charge configurations. The Coulomb force on any charge q at a point is just $\mathbf{F}_c = q\mathbf{E}$ where **E** is evaluated at that point.

A small charge q is used to explore the field so that it has a minimal effect upon the object it is measuring. Suppose we let q ap-

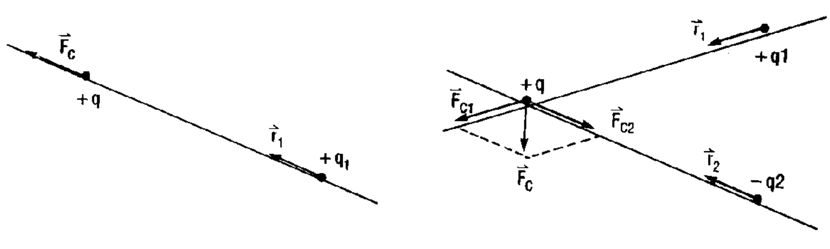

FIG. 1—COULOMB'S EXPERIMENTS showed that a static electric charge produces a force F_c on another charge. A positive-point charge $+q_1$ produces a force on another positive charge $+q$ in the direction of the unit vector r_1 (*a*). A positive charge $+q_1$ produces a force F_{c1} on $+q$ in the direction of the unit vector r_1. A negative charge $-q_2$ produces a force F_{c2} directed opposite to r_2. The total force on $+q$ is the vector sum F_c (*b*).

Gaussian surface enclosing a charge q, as in Fig. 5-*a*. Divide the surface into an infinite number of infinitesimal surface areas **ds**. Area is a vector because it has a magnitude and also a direction, or orientation in space which is taken as normal (perpendicular) to the surface, and pointing outward away from the enclosed volume. Each infinitesimal area is essentially a small plane with an **E** vector through it. Because the surface is arbitrary, each **ds** and its **E** vector does not have to be parallel. In other words, **E** may not be normal to the plane.

To find the apparent outflow, we need to consider only the component of **E** normal to the plane; the rest is just flowing over the surface. The scaler, or dot product, **E·ds**, does that by giving the product of the magnitude of **E** parallel with **ds** times the magnitude of **ds**. That is the same as the product of the magnitude of the effective area (the projected area with **ds** parallel to **E**) times the magnitude of **E**. The apparent flow is electric flux. Summing the contributions from each **ds** over the entire surface gives the total flux

$$\psi = \int \mathbf{E} \cdot d\mathbf{s} \text{ (N/C m}^2\text{)}$$

ψ is proportional to the charge q within the volume since **E** is proportional to q. Because **E** obeys the $1/r^2$ law, and the effective area obeys the r^2 law, ψ is independent of the surface. If a number of point charges were contained inside the volume, ψ would be proportional to the total charge because the total **E** field is the linear superposition of their **E** fields. The proportionality constant is $1/\epsilon_0$, therefore

$$\psi = q/\epsilon_0$$

Charges outside the volume would not contribute to the **E** field. The reason for that is if some **E** came in through some **ds**'s, it would go out through some other **ds**'s in just the right amounts to cancel out because of the $1/r^2$ and r^2 dependence. Graphically, lines having the direction of **E** at each point, and with their closeness proportional to ψ are sometimes used to depict the **E** field. That's a convenient approach, but it must be remembered that the **E** field is actually a vector at each point in space.

If the Gaussian surface shrinks down to a point, then all the **ds**'s would shrink to zero and so would flux ψ. The ratio of the change in flux to the change in volume as the surface shrinks reaches a limiting value. That limiting value is called the divergence, and is symbolized by

$$\nabla \cdot \mathbf{E} = d\psi/d_{volume}$$

That must be proportional to the charge per unit volume

$$dq/d_{volume} = \rho,$$

which is called charge density within the surface, therefore

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0 \text{ (N/C m)}$$

Since ϵ_0 is a constant and is independent of the volume, the above equation could be written as

$$\nabla \cdot \epsilon_0 \mathbf{E} = \rho$$

A number of **E** field instruments (small +q's) scattered around a region, would diverge away from a positive charge (a positive divergence) or converge upon a negative charge (a negative divergence). A field with zero divergence cannot start or end at the point.

The apparent rotation of the **E** field around a point can be measured by imagining an arbitrary closed curve, called an amperean loop, of length **l**, encircling a charge q as in Fig. 5-*b*. Divide the loop into an infinite number of infinitesimally small lengths, **dl**. The direction of **dl** is taken as counter-clock wise. A loop is used because the **dl**'s define general directions around q, whereas for a surface, the **ds**'s define general directions away from q. Each **dl** is so small that it is essentially a straight line segment with an **E** vector through it. The apparent rotation at each **dl** is the magnitude of the component of **E**, parallel to **dl** times the magnitude of **dl**. We must again use the dot product **E·dl** to allow for the fact that **E** may not be parallel to **dl**. That gives the magnitude of the **E** component parallel to **dl** times the magnitude of **dl**.

Imagine moving around the loop, summing up **E·dl** to obtain the total apparent rotation, or electric circulation. Since **E** points radially along **r**, the only place **E·dl** is non-zero is where **dl** has a component parallel to **r**. But the entire loop is closed, so for any amount it moves out radially, it must at some place move that same amount inward. The field is symmetrical, therefore whenever **E·dl** is positive along some **dl**'s, it is negative by the same amount along other **dl**'s, with a net result of zero. In calculus notation

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

The circle on the integration symbol reminds us that the loop is closed. Again, by linear superposition, that is true for any static charge configuration.

If the amperean loop shrinks down to a point, all the **dl**'s would shrink to zero, and so would $\int \mathbf{E} \cdot d\mathbf{l}$ (even if it weren't already zero). But the ratio of the change in $\oint \mathbf{E} \cdot d\mathbf{l}$ to the change in the enclosed area as the loop shrinks reaches a limiting value. That limiting value is called the curl, and is symbolized by

$$\nabla \times \mathbf{E} = d(\oint \mathbf{E} \cdot d\mathbf{l})/d_{area}.$$

The curl is a vector, since area is a vector. It's direction is taken as

continued on page 79

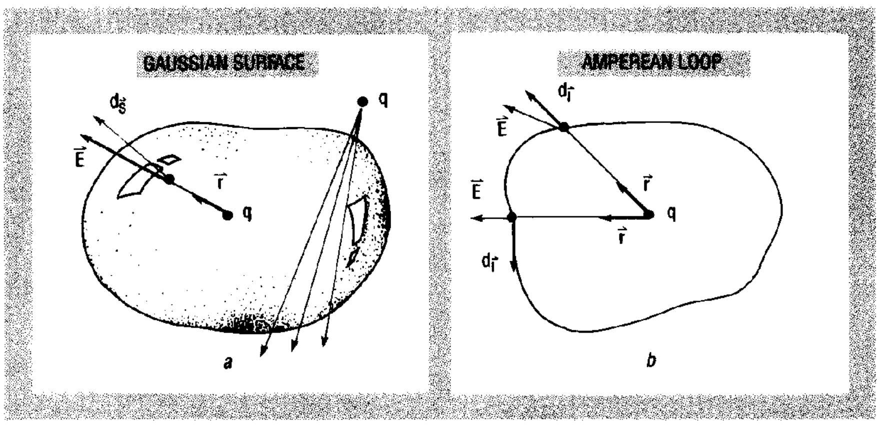

FIG. 5—CHARACTERISTIC OF AN E FIELD. In (*a*) a Gaussian surface composed of an infinite number of infinitesimal areas d_s surrounds a positive charge q. The total apparent flow of the electric field and the electric flux is the sum of E·d_s over the entire surface, which is proportional to q. Flux from charges outside the surface does not contribute because whatever flux "flows" through the surface must also flow back out. In (*b*), an amperean loop composed of an infinite number of infinitesimal lengths, dl, encircles the charge. The electric circulation around the loop $\int E \cdot d_l$ is zero.

values. If ∇h is non-zero at a point, then the neighboring points at right angles to ∇h have the same value h.

For example, imagine standing at a point on a hillside with the height, h, at every point known. ∇h would point in the direction of maximum increase in h, and the maximum decrease would be in the opposite direction, -∇h. If you walked at right angles to ∇h at each point, you would walk along a level or equi-height line. If ∇h equals zero, you would be at a flat spot. ∇h is a vector field since it gives a vector for each point.

Vector fields can be characterized by the fact that they give the impression of flow, as shown in Fig. 4-*a*–*e*. In general, near any point the apparent flow diverges away from (or toward) the point, rotates or curls around a point, or is a combination of both. If the field describes a material, such as air velocity, then there is an actual flow of material.

To measure the apparent flow, or spreading out of the **E** field from a point, imagine an arbitrary closed surface, called a

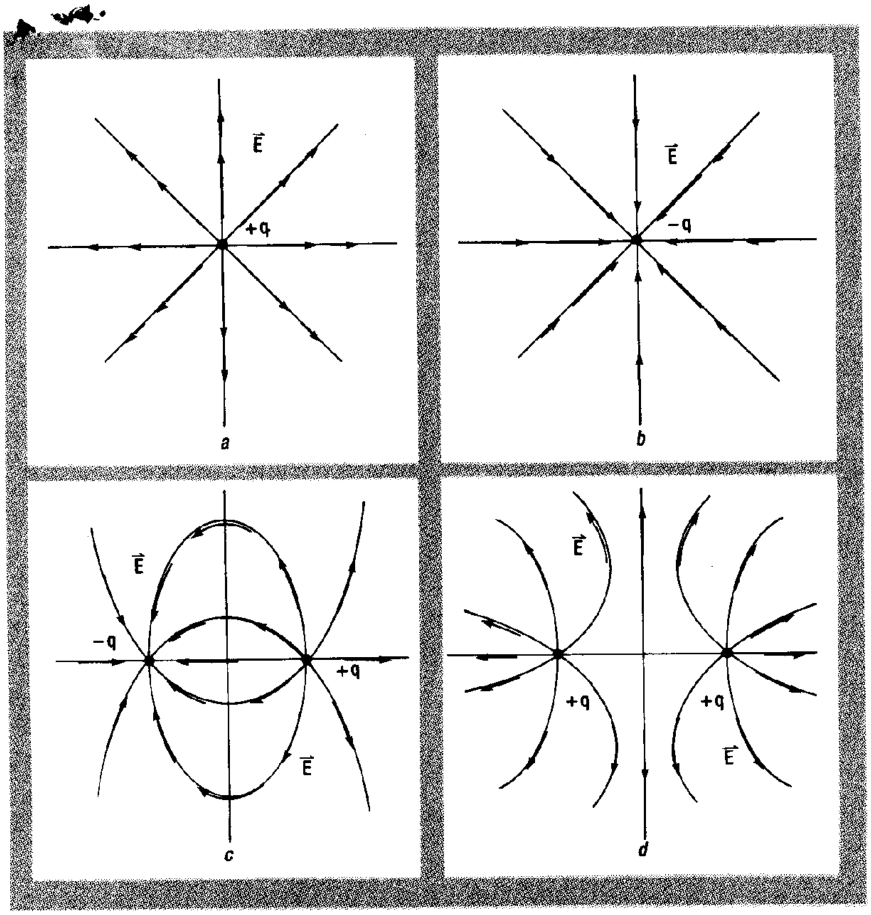

FIG. 2—THE E FIELD IS A RESULT OF the forces between static electric charges. Field vectors are shown in a cross section of a 3-dimensional space for a static positive-point charge (*a*) and for a static negative-point charge (*b*). In (*c*) and (*d*) the E fields for two static charges are shown; the vectors are located at their tail points.

proach 0. In reality, we can't vary the charge continuously since charge appears to come in multiples of e, but we can idealize the process. The force felt by that charge will decrease as the charge decreases, but the ratio of the change in force to the change in charge will reach some limiting value. That relationship is written in the calculus notation

$$E = \lim_{\Delta q \to 0} \frac{\Delta F}{\Delta q} = \frac{dF}{dq}$$

Very small positive point charges (so small that their **E** fields can be neglected) can be thought of as ideal devices to explore the **E** field.

Field characteristics

A scaler field, as shown in Fig. 3, can be characterized by the fact that a scaler value can change by a certain amount in a particular direction. In any real field, the values differ little from one point to neighboring points. The gradient of a scaler field is a mathematical operation. It gives a vector that points in the direction for which the value undergoes the largest change, and whose magnitude is that rate of change. The gradient of the scaler field h is symbolized by ∇h. If ∇h equals zero, then the neighboring points must all equal h

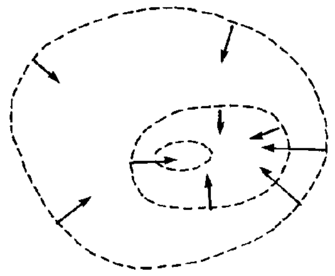

FIG. 3—THE GRADIENT OF A SCALER FIELD is a vector field. The scaler value is the same along each dashed line called an equi-line. Each of the vectors have a magnitude proportional to the greatest rate of change in scaler value per unit distance, and point in the direction of the greatest change. The vectors are perpendicular to the equi-line at their respective points.

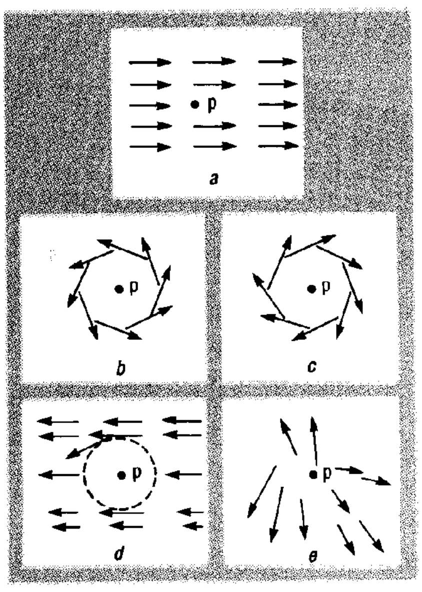

FIG. 4—VECTOR FIELDS GIVE the impression of flow that diverges from, or curls around, an arbitrary point p. Both the divergence and curl of the field are zero in (*a*). There is zero divergence and non-zero curl in (*b*); the curl is a vector out of the page at the point. In (*c*), the direction is reversed, and the vector points into the page. In (*d*) there is zero divergence but non-zero curl since there are non-symmetrical contributions around the closed line. Both the divergence and curl are non-zero in (*e*); these fields could not be static E fields.

ELECTROMAGNETICS

continued from page 68

the direction of the extended thumb of the right hand with the fingers wrapped in the general direction taken around the loop. In the case of the static **E** field

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0,$$

therefore

$$\nabla \times \mathbf{E} = 0.$$

The curl is a vector measure of the apparent rotation of the field about a point. If a number of **E** field instruments were scattered around a region, the group would not rotate.

The divergence and curl of the types of fields we're discussing completely characterize the field; **E** can be found if $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$ are known. This is known as Helmholtz's theorem.

The curl of a vector field is always zero, if, and only if the field is the gradient of some scaler field. Consider our h field example. If $\nabla \times \nabla h$ were non-zero, then in following a closed path from some point and back to the beginning, one encounters different rates of change of height times distance when taking different paths. $\oint \nabla h \cdot d\mathbf{l}$ would be path dependent. That would amount to leaving from a point at, for instance, 50 meters in elevation and returning only to find the elevation is 300 meters, or 2 meters, depending upon what path was taken!

The divergence of a field is always zero only if the field is the curl of another field. Imagine the fields of Fig. 4 in 3-dimensional space. Curl the right-hand fingers in the direction of the apparent rotation around the point. The extended thumb is the direction of the curl vector at that point. Conversely, consider the vectors shown as curl vectors. Direct the thumb along them and the fingers will curl in the direction of the field vectors. The field vectors seem to cancel, and not spread out. Those fields are the curl of another vector field. Try that with Figs. 2 or 4 and you'll get conflicting results.

Next time, we'll develop Ohm's law and look at the **E** field in materials, which will provide further insight into Maxwell's equations. **R-E**

A NEW FORCE?

A very important concept was brought out in the June 1976 article about Golka and "12-Million Volts." The idea of "re-creating the past to solve future needs" caught my attention.

Over 150 years ago, Oersted discovered a tiny force that causes a permanent magnet to turn at right angles to a current-carrying conductor. He didn't know it but he could have substituted an iron wire for the compass needle although he would not have been able to determine North from South. In all this time we have not been able to produce a permanent magnet that has a magnetic field similar to the one around a current-carrying conductor.

Anytime we find a new force, no matter how tiny, big things begin to happen. Modify Oersted's experiment. Obtain a straight iron-wire 12-inches long that has the same diameter as a current-carrying conductor and suspend it 1/16-inch below and in parallel with the current-carrying conductor with three loops of thread. No matter which way the current flows in the current-carrying conductor, the iron wire is always lifted. How can we describe and use this new force?

We know if current flows in the same direction in two parallel conductors, they attract each other. Reverse the current in one conductor and they repel each other. There is no current in the iron wire yet it is always attracted. Why?

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