A FOURTH-ORDER STATE VARIABLE FILTER FOR
LINKWITZ-RILEY ACTIVE CROSSOVER DESIGNS

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ABSTRACT

A new fourth-order state variable active filter is presented with complete design equations. Sensitivity analysis is included for the effects of component tolerances. An application design example is given for a Linkwitz-Riley crossover network. Where multiple crossover points are required, it is shown that the state variable approach affords the most economical solution, offering simultaneous adjustment of both the high and low pass filters with minimum parts count.

INTRODUCTION

While much work has been done and published on 2nd-order state variable active filters, little, if any, has reached print on how to directly implement a 4th-order state variable filter. The most common approach would be to cascade two 2nd-order circuits together. This obviously works, but if the requirement exists for simultaneous 4th-order high pass and low pass outputs, then at least three 2nd-order sections are required, and if the corner frequencies must be easily adjustable, then the circuit becomes unnecessarily complex.

The following development yields a true 4th-order state variable active filter with simultaneous high pass and low pass outputs that is easily made adjustable. Equations are developed and a Linkwitz-Riley (1) crossover design is implemented as an application example.

GENERAL EQUATION

The topology is a straightforward extension of the normal 2nd-order design (2), obtained by adding two more integrators and feedback paths, and appears as Figure 1.

Working the La Place domain, the transfer function, $T(s)$, of each integrator is designated as

$$T(s) = - \frac{1}{RCS} \tag{1}$$

Two equations can be immediately written for the summing junction currents at the positive and negative inputs of Z1. After manipulating and substitution of the identical integrator transfer functions into these equations, they can be combined and solved to produce one general transfer equation relating V_L to V_i as follows:

$$\frac{V_L}{V_i} = \frac{\dfrac{R4R6K2}{R3R5R7K1(RC)^4}}{s^4 + \dfrac{R1R6K2}{R3R5R7K1(RC)}s^3 + \dfrac{R2}{R5(RC)^2}s^2 + \dfrac{R1R4K2}{R3R5R7K1(RC)^3}s + \dfrac{R2}{R7(RC)^4}} \tag{2}$$

where $\qquad K1 = R4R6 + R1R6 + R1R4$

and $\qquad K2 = R3R5R7 + R2R3R7 + R2R3R5 + R2R5R7$

It is also noted that

$$V_L = \frac{V_H}{(RCS)^4} \tag{3}$$

The general equation for a 2nd-order transfer function is

$$s^2 + a\omega_n S + \omega_n^2 \tag{4}$$

where $\qquad Q = \frac{1}{a}$

A 4th-order transfer function can always be reduced to the product of two 2nd-order functions; therefore, a general 4th-order equation can be written by squaring (4) (and allowing for different a terms) as follows:

$$S^4 + (a_1 + a_2)\omega_n S^3 + (2 + a_1 a_2)\omega_n^2 S^2 + (a_1 + a_2)\omega_n^3 S + \omega_n^4 \tag{5}$$

Equation (5) for two identical 2nd-order terms becomes

$$S^4 + 2a\omega_n S^3 + (2+a^2)\omega_n^2 S^2 + 2a\omega_n^3 S + \omega_n^4 \qquad (6)$$

Equating the coefficients of the denominator of equation (2) with either equations (5) or (6) produces the following equalities

$$R2=R7 \qquad (7)$$

$$R4=R6 \qquad (8)$$

and

$$\omega_n = \frac{1}{RC} \qquad (9)$$

Since equation (2) is a low pass filter, the gain expression, H_O, can be found by letting S=O and simplifying to

$$H_O = \frac{R4(2R3R5+R2R3+R2R5)}{R3R5(R4+2R1)} \qquad (10)$$

The quality factor, Q, of the overall 4th-order expression is a function of the Q of each of the 2nd-order transfer functions and is found by equating coefficients with equation (5). It is

$$Q = \frac{Q_1 Q_2}{Q_1 + Q_2} = \frac{R3R5(R4+2R1)}{R1(2R3R5+R2R3+R2R5)} \qquad (11)$$

For two identical 2nd-order terms, the coefficients of equation (6) are used, and equation (11) reduces to

$$Q = \frac{2R3R5(R4+2R1)}{R1(2R3R5+R2R3+R2R5)} \qquad (12)$$

At this point, it is necessary to know the desired 4th-order expression in order to proceed further. Once known, the exact expressions for each resistor value can be solved by equating coefficients. As an example of this procedure and to show a real-world application for Figure 1, the remainder of this paper will deal with implementing a Linkwitz-Riley active crossover.

LINKWITZ-RILEY CROSSOVER

A Linkwitz-Riley crossover design consists of two identical Butterworth 2nd-order functions in cascade. By letting ω_n =1, the normalized expression for each can be written as

$$S^2 + \sqrt{2}\, S + 1 \qquad (13)$$

And the resultant 4th-order response becomes

$$S^4 + 2\sqrt{2}\, S^3 + 4S^2 + 2\sqrt{2}\, S + 1 \qquad (14)$$

Adding that it is desired for H_O=1, and equating coefficients, yields the following relationships:

$$R1 = 2\sqrt{2}\ \ R4 = 2\sqrt{2}\ \ R6 \qquad (15)$$

$$R2 = (4\sqrt{2}\ -5)R3 = 4R5 = R7 \qquad (16)$$

It is also useful to note that for the given conditions

$$R6 = \frac{R1Q}{2H_O} \qquad (17)$$

There is no unique solution for a problem with this many variables so at least one value may be arbitrarily chosen. Probably the most useful is to let

$$R1 = R2 \qquad (18)$$

Resistor, R1, is selected for the desired input impedance, and all other values are calculated from it. Figure 2 shows the circuit with each relationship indicated.

SENSITIVITIES

Using equations (10) and (12) and the Linkwitz-Riley resistor relationships, the sensitivities of H_O and Q to variations from ideal resistor values can be calculated. The results become a useful guideline for predicting non-ideal behavior. The numbers given have been rounded off to the nearest whole ratio for convenience.

From equation (10), the sensitivities for the gain term, H_O, are

$$S_{R1}^{H_O} = -S_{R4}^{H_O} = -S_{R6}^{H_O} = -\frac{6}{7} \qquad (19)$$

$$S_{R2}^{H_O} = -S_{R5}^{H_O} = S_{R7}^{H_O} = \frac{2}{3} \qquad (20)$$

$$S_{R3}^{H_O} = -\frac{1}{10} \qquad (21)$$

And from equation (11), the sensitivities for the quality factor, Q, are

$$S_{R1}^{Q} = -S_{R4}^{Q} = -S_{R6}^{Q} = -\frac{3}{20} \qquad (22)$$

$$S_{R2}^{Q} = -S_{R5}^{Q} = S_{R7}^{Q} = -\frac{2}{3} \qquad (23)$$

$$S_{R3}^{Q} = \frac{1}{10} \qquad (24)$$

From equation (19)-(21), it is seen that gain variations will be most susceptible to errors in R1, R4 and R6 and least affected by resistor, R3. While, equation (22)-(24) show Q anomalies will be due mainly to resistors, R2, R5, and R7.

ADJUSTABLE CORNER FREQUENCIES

Making the low pass and high pass corner frequencies adjustable can be done in several ways. The simplest method is to make all of the integrator resistors variable. This can be done with a single 4-gang potentiometer. While producing a minimum parts count circuit, there are several things that must be carefully considered before committing the final design to a single potentiometer approach.

The first is cost. A quality, precision 4-gang potentiometer can be expensive. Since most applications for potentiometers require only single or double sections, there is no large volume use of 4-ganged units, which would help make availability better and cost lower.

The second area of concern is the absolute value of each section relative to each other. Most potentiometers (even good ones) have a 10-20% tolerance on absolute value, so the difference between each section can be as much as 40%. This may necessitate a separate trim resistor for each integrator.

The last potential trouble area with 4-ganged units is tracking. Tracking is the relative difference between each section for any given angle of rotation. Tracking specifications from section to section must be reasonably tight or amplitude errors will show up in the passband.

Another approach to the problem of corner frequency adjustability is to use fixed resistors selected by FET switches. Such an arrangement appears as Figure 3. In Figure 3A, a 2-bit (as in digital, not price!) binary switch is used to control one-half of a CMOS 4052 dual 4-channel analog multiplexer, to select one of four integrator resistors.

In Figure 3B, the concept is expanded to eight input resistors by using a 3-bit binary switch and a CMOS 4051 8-channel analog multiplexer.

The technique can be increased to 16 resistors by employing (2) 4051's, an inverter controlling their inhibit ports, and a 4-bit binary switch, as outlined in Figure 3C.

Several immediate advantages can be seen with the multiplexer approach. First is the accuracy and matching of the four integrator resistors. By making the resistor values large compared to the "on" resistance of the CMOS switches, the error introduced by R_{ds} (on) can be effectively eliminated. The matching, using 1% resistors, is normally more than adequate, although the option remains of using even tighter tolerance should the application require it.

Another benefit is the repeatability of settings. The binary switch does not have the ambiguity of trying to position the potentiometer to the same place each time. But there is a disadvantage also. With the switch approach, only those frequencies are possible; where, with the potentiometer, the user has continuous choice of any frequency between its two extremes. Only the exact application can decide this one.

The parts cost trade-off of the binary switch approach is surprising. It is actually lower than the 4-gang potentiometer; but, here again there is a compromise. The parts count goes up drastically, so the overall out-the-door cost may be higher. For a typical crossover design requiring 8 positions the parts difference just for the switching option is about 40 to 1!

Along with the increase in the number of required parts, goes a decrease in overall reliability. The old saying is still very true and must be evaluated carefully before committing to a final design-- "Two parts are less reliable than one".

SUMMARY

A new topology has been presented for a 4th-order state-variable active filter with simultaneous high pass and low pass outputs. The general transfer function has been developed and reduced. A specific application for a Linkwitz-Riley crossover design has been shown along with exact design equations. And a discussion of the designs and tradeoffs of making the corner frequencies variable was presented.

It has been shown that with the new 4th-order state variable filter and equations presented, more cost-effective 4th-order low pass and high pass designs can be realized. It is no longer necessary to cascade two 2nd-order state variable filters to obtain the high pass output and then add a third 2nd-order circuit to get the low pass.

An example of a Linkwitz-Riley active crossover has been given to show the ease of use of the single 4th-order approach. A discussion of the many trade-offs involved in making the corner frequencies variable has been included for reference and further study. The single 4-gang potentiometer design offers a minimum parts-count solution with higher parts cost along with potential tracking and repeatability errors, while the analog switch approach gives the best accuracy and repeatability with the price of increased circuit complexity.

REFERENCES

1. S.H. Linkwitz, "Active Crossover Networks for Non-Coincident Drivers," J. Audio Eng. Soc., vol. 24, pp. 2-8 (Jan./Feb. 1976).

2. W. J. Kerwin, L.P. Huelsman, and R.W. Newcomb, "State Variable Synthesis for Insensitive Integrated Circuit Transfer Functions," IEEE J. Solid-State Circuits, SC2, pp. 87-92 (Sep. 1967).
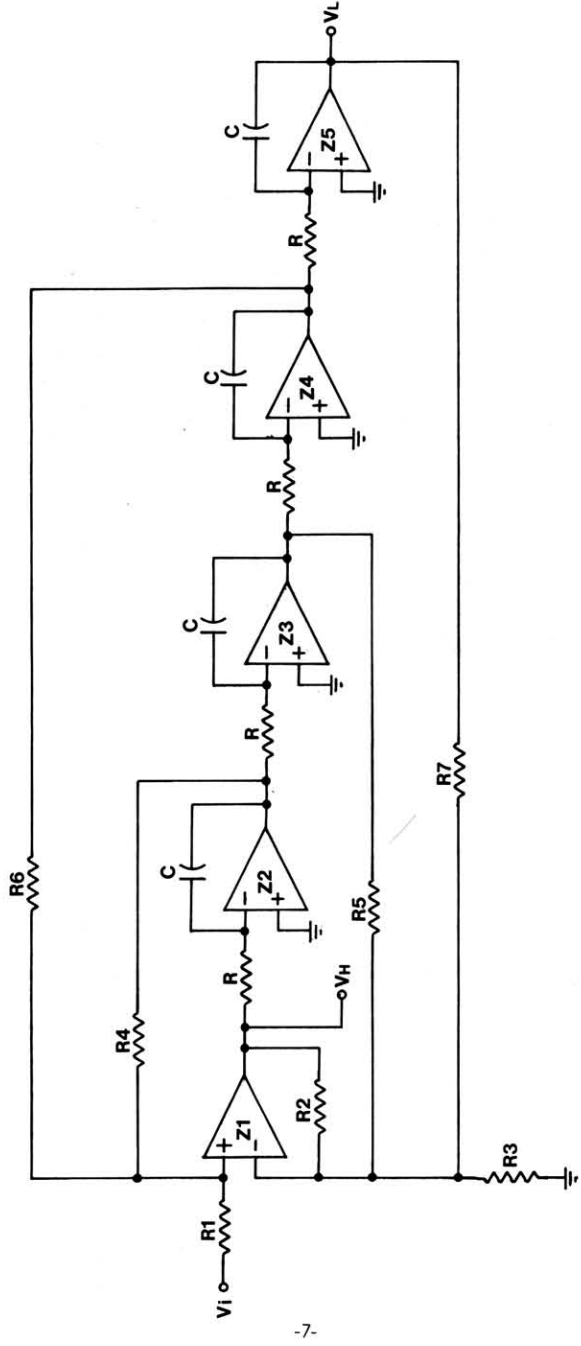
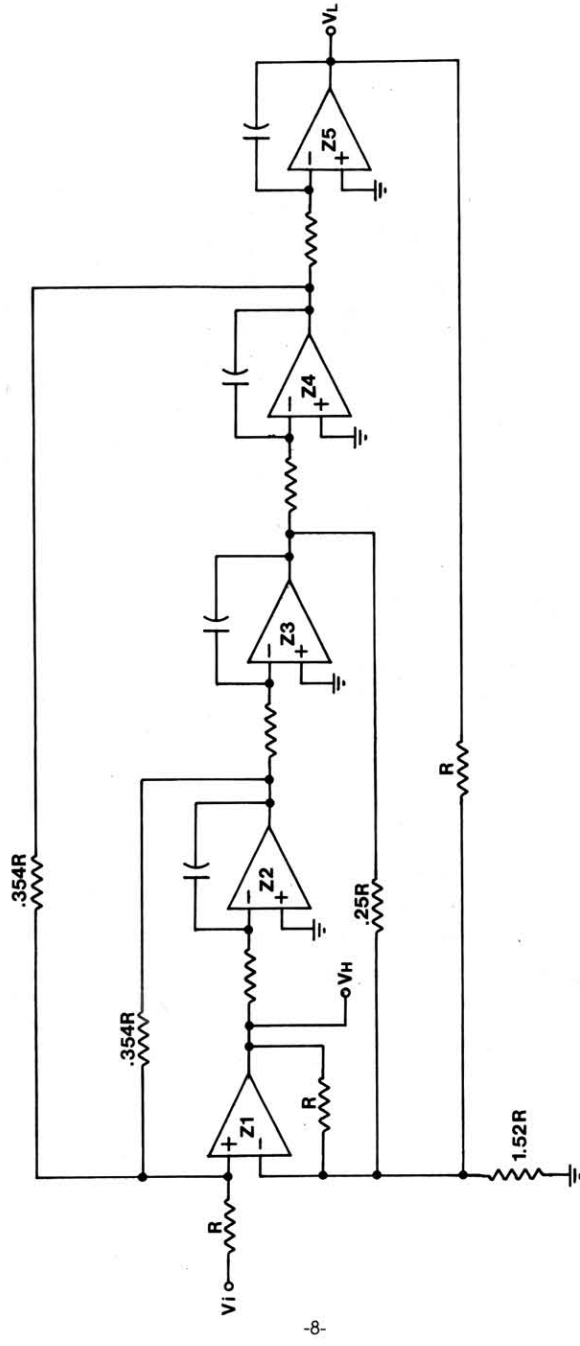Figure 1.   General Purpose 4th-order State Variable Filter

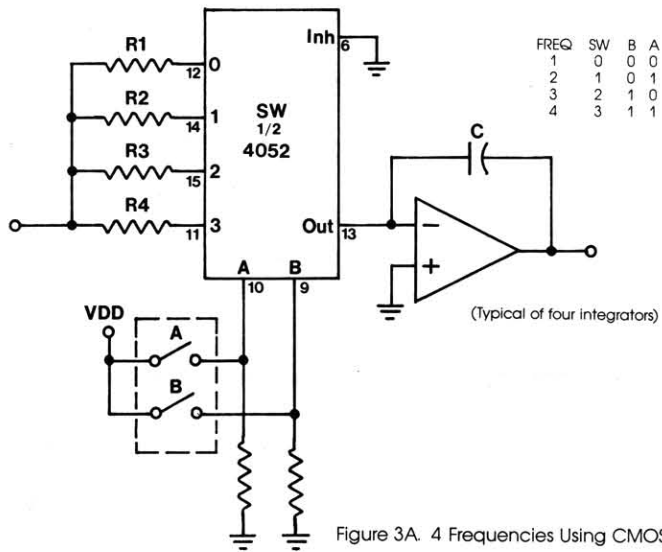Figure 2.   Resistor Relationships for a Linkwitz-Riley Crossover
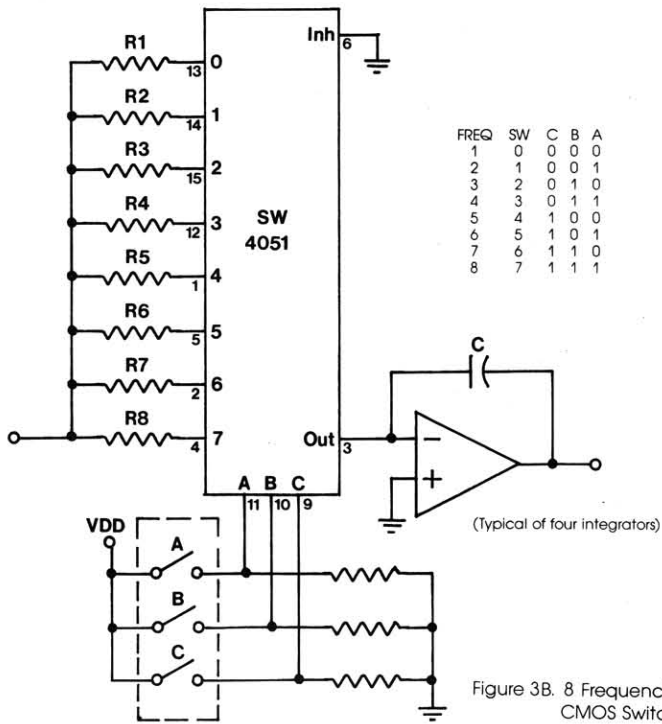
Figure 3A. 4 Frequencies Using CMOS Switches

FREQ	SW	B	A
1	0	0	0
2	1	0	1
3	2	1	0
4	3	1	1

(Typical of four integrators)

FREQ	SW	C	B	A
1	0	0	0	0
2	1	0	0	1
3	2	0	1	0
4	3	0	1	1
5	4	1	0	0
6	5	1	0	1
7	6	1	1	0
8	7	1	1	1

(Typical of four integrators)

Figure 3B. 8 Frequencies Using CMOS Switches

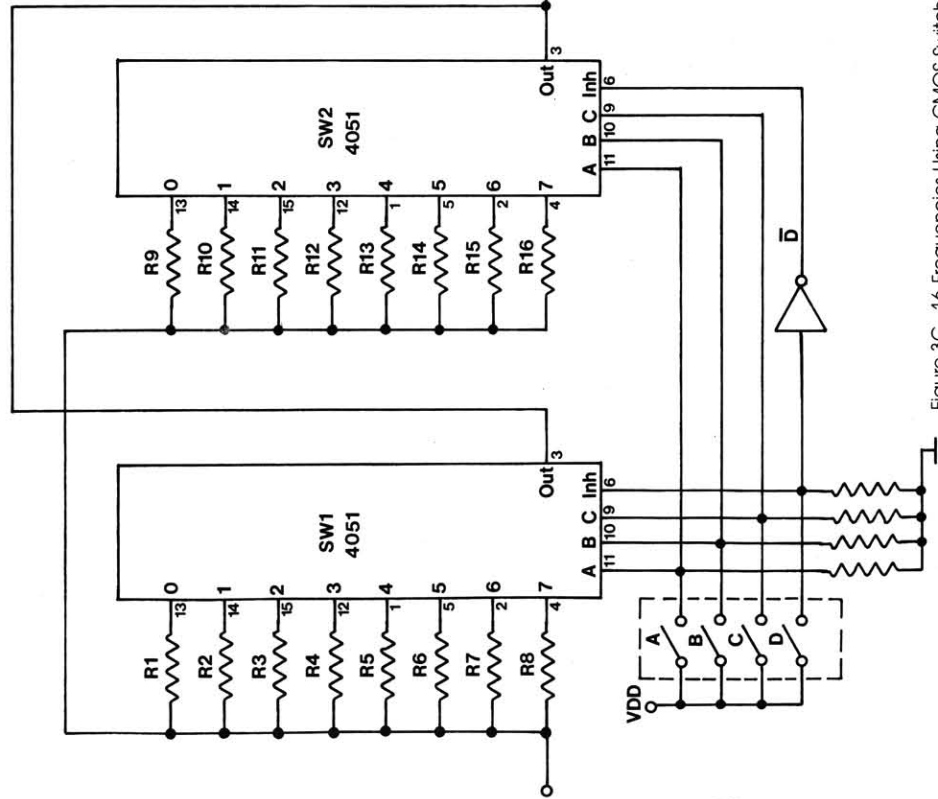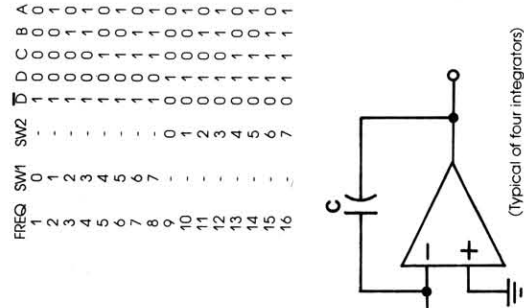| FREQ	SW1	SW2	D	C	B	A
1	0	-	0	0	0	0
2	1	-	0	0	0	1
3	2	-	0	0	1	0
4	3	-	0	0	1	1
5	4	-	0	1	0	0
6	5	-	0	1	0	1
7	6	-	0	1	1	0
8	7	-	0	1	1	1
9	-	0	1	0	0	0
10	-	1	1	0	0	1
11	-	2	1	0	1	0
12	-	3	1	0	1	1
13	-	4	1	1	0	0
14	-	5	1	1	0	1
15	-	6	1	1	1	0
16	-	7	1	1	1	1

(Typical of four integrators)

Figure 3C. 16 Frequencies Using CMOS Switches