Understanding Sound System Design and Feedback Using (Ugh!) Math

by Rick Frank

Shure Incorporated
222 Hartrey Avenue
Evanston, Illinois 60202-3696
(847) 866-2200

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One of the basic building blocks of sound system design is the Potential Acoustic Gain (PAG) equation. With a few simple calculations it can provide a guide to dealing with the problems and restrictions encountered in this process.

Here's a common situation confronting a sound contractor trying to provide a system for an existing room. He has a good working knowledge of the audio chain and the products at his disposal to assemble a "working system." He knows the room—let's say a conference room. And he knows where he (or the client) would like to place the microphones and speakers. But how can he know in advance whether the components he would like to install, and the layout he would like to use, will produce the gain needed without feedback? And for that matter, how much gain is needed?

If he wants to do more than try what he "feels" should work and then hope for the best, he will pull out his calculator, plug in the dimensions of his system design, and let the equation guide him in the right direction.

Unfortunately, many people who could benefit by understanding and using the equation are put off by its apparent complexity. This doesn't have to be the case as I will show in the following pages. A calculator with a "log" button and possibly a sound level meter are really all you need to take some of the mystery and potential disaster out of sound system design. It all starts with the Inverse Square Law.

THE INVERSE SQUARE LAW—EASIER THAN IT SOUNDS

Have you noticed that as you walk away from someone speaking, the talker's voice level decreases as perceived by your ears? What's happening is this: distance from a sound source affects the sound pressure level (SPL) on your ears in a particular way. It's described by the *inverse square law* which states that as listeners double their distance from the sound source, the SPL they perceive will decrease by 6.02dB. So if you first stand 4 feet from a talker and then move to a new position 8 feet from the talker, you should notice about a 6.02dB drop in level. (A one dB increase is barely audible, 3dB is a generally noticeable change, and a 10dB increase is considered to be twice as loud.) Mathematically the inverse square law looks like this:

New Level = Old Level + 20 X log of old distance - 20 X log of new distance
or in math shorthand

$$L' = L + 20 \log D - 20 \log D'$$

This equation describes what happens in a so-called "free field." That means that there is no interference from things like reflected sounds and that the origin of the sound is a point source (a sound source which has much smaller dimensions than the distance to the listener). Since even the earth can complicate the issue, there is almost never a situation where we encounter the pure effects of this equation.

That doesn't mean we have to abandon the theory, but rather that we have to use it with the understanding that it will not perfectly predict what is going to happen when we apply it to a real situation. Because it's easier to understand this way, let's try working through an example by assuming we're in a free field.

Using the previous example with the old distance, D, equal to 4 feet and the new distance, D' ("D prime"), equal to 8 feet, you can use your calculator to test this free space law. Let's say that the old level, L, of the talker measures 70dB at 4 feet. Incidentally, I've always found it is easiest to work in reverse on calculator logarithm (log) computations. So to work through the problem, punch the D' quantity "8" into the calculator, find the logarithm (hit the log button), and get 0.90309.

Log of 8 = 0.90309

Now multiply by 20 and get 18 (rounded from 18.0618). (Since 1dB is considered to be the smallest difference we can actually hear, I'll round all these figures to the nearest whole number.)

0.90309 x 20 = 18 (rounded from 18.0618)

Save this figure and punch in the old distance, D, of "4". If you follow the same steps the new answer is 12 (again, rounded).

log of 4 = 0.6021

0.6021 x 20 = 12 (rounded from 12.0412)

The final answer is just the arithmetic of the inverse square law: take the old sound pressure level of 70 plus 12, minus 18, to arrive at the new level, 64 — 6dB less than the old level.

New Level = Old Level + 20 X log of old distance - 20 X log of new distance

64 = 70 + 12 - 18

Because of the nature of logarithms and the inverse square law, this solution will occur with any pair of distances where one is twice the other. Let's say we move from 13 feet to 26 feet away from the talker. Working backwards again, entering 26 followed by the log button and multiplying by 20 gives 28dB (rounded). The same procedure with 13 gives 22dB. And the arithmetic (70 plus 22 minus 28) results in a level of 64dB — the same 6dB drop as we had with 4' to 8'.

log of 13 = 1.1139434,

1.1139434 x 20 = 22 (rounded from 22.278867),

log of 26 = 1.4149733, 1.4149733 x 20 = 28

New Level = Old Level + 20 X log of old distance - 20 X log of new distance

64 = 70 + 22 - 28

Of course this works in reverse, as your ears will tell you. If we move closer to the sound source by cutting the distance in half (from 8' to 4'), the sound level rises by 6dB. ***The important point to remember is that in order to get a significant 6dB sound level change, the distance must change by double or half.***

CRITICAL DISTANCE IS CRITICAL

A lot of the limitations on the practical use of the inverse square law and the PAG equation have to do with what's called "critical distance." Basically, when the listener is within the critical distance of the source, the inverse square law works pretty well. Outside the critical distance things get much more complex.

The term "critical distance" is defined as the distance from a sound source where the direct sound is the same level as the reverberant sound. With a sound level meter and a de-tuned FM radio you can make a good estimate of critical distance in a room. The radio serves as a sound source to provide a constant, broad spectrum sound. If you walk backwards from the sound source with a sound level meter, you will reach a point where the level stops registering change on the meter. At that point, if you walk back *towards* the sound source until the meter increases exactly 3dB you will have reached the critical distance. (Equal sound from the direct source and the reverberant field add up to 3dB more than the reverberant sound alone.)

Whether or not you actually make estimates of the critical distances within a system, there is one thing that is important to know. Typically in sound systems, the distance between the microphone and the talker is the only place where the system component receiver (the listener or microphone) is less than the critical distance from the source. Later when we look at the actual distances between the parts of a system, we'll see the dramatic effects of changing this distance.

HOW MUCH ACOUSTIC GAIN IS NEEDED?

With this in mind we're ready to look at the question of how much gain is needed for listeners in a given room. (Remember that in a real room the reflective surfaces create echoes and a reverberant field that interfere with the theoretical results of the inverse square law as well as the potential acoustic gain equation. This interference generally *lowers* the actual performance of a system.)

Let's take an example of a conference room with a 22' table. Normal conversational level in a quiet room at a normal conversational distance of 2 feet is about 70dB. The inverse square law will tell us how much the loss will be if the listener is at the opposite end of the table, 22 feet away. That listener would ideally like to hear exactly what the close listener hears. The distance to the far listener creates the loss which the sound system should replace.

Putting the figures from this scene into the inverse square law equation (using the mathematical shorthand) looks like this:

$$L' = L + 20 \log D - 20 \log D'$$

$$= 70 + 20 \times \log 2 - 20 \times \log 22$$

Working backwards from the new distance, 22, punch in 22, take its log and multiply the result by 20. Write down that result (27), take the log of 2 the same way and multiply it by 20, and you get 6. Now the arithmetic: 70 plus 6 minus 27 equals 49 — telling us that the distant listener experiences a loss of 21dB (70 - 49).

$$L' = L + 20 \log D - 20 \log D'$$

$$49 = 70 + 6 - 27$$

$$49 = 70 - 21$$

$$49 - 70 = -21 \text{ (a loss of 21dB)}$$

This is the sound system's target. The sound system must provide at least 21dB of gain for the farthest listener to hear the talker as if she is 2 feet away. This is called the needed acoustic gain (NAG). If the distant listener is supposed to hear as well as the near listener then the system must make up the 21dB loss caused by the distance.

Of course if you have a sound level meter you can take a reading in both the near and far positions and subtract to get to this point. But with the first method you get to understand and see the inverse square law in action. It may also be interesting to compare the two answers to see how much the room acoustics affect the results that arc predicted by the inverse square law.

HOW MUCH ACOUSTIC GAIN CAN THE SYSTEM PRODUCE?

Now let's work with the Potential Acoustic Gain equation to see if our system will theoretically produce the acoustic gain we need before it feeds back. To start we need to look at some of its individual terms. Each of these terms is a shorthand for a basic distance (D) in the sound system design, as illustrated in Figure 1. Since there are four significant distances between the elements in the system, each "D" has a subscript such as "D_1", "D_S", ("D sub one", "D sub S") etc. Specifically, this is what each term stands for:

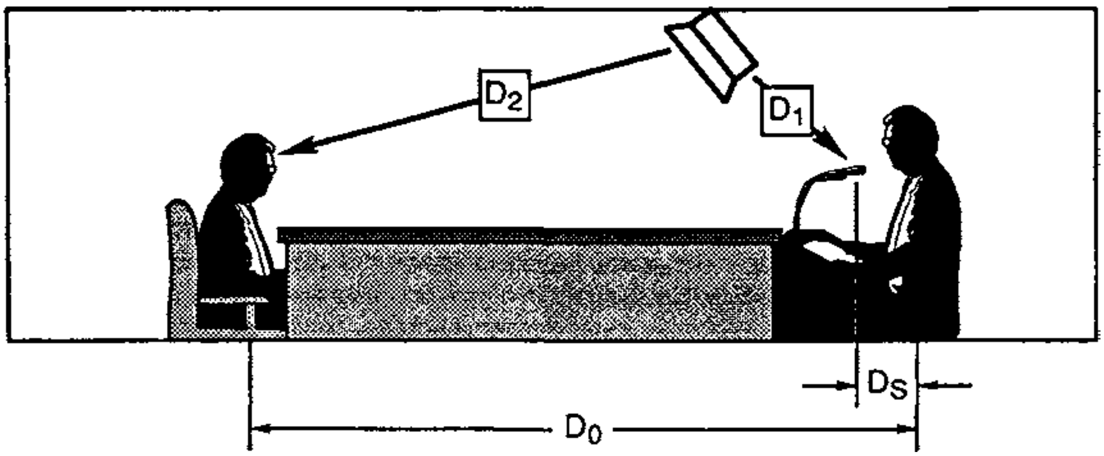

Figure 1. The basic distances in a simple sound reinforcement system.

"Electronic to Electronic" or "People to People" Distances

- D_1 is the distance between the microphone and the loudspeaker (remember "1" as the first pan of the system).
- D_0 is the distance between the talker and the farthest listener (remember "0" for observer).

"People to Electronic" Distances

- D_2 is the distance between the loudspeaker and the farthest listener (remember "2" as the last pan of the system).
- D_S is the distance between the talker and the microphone (remember "S" for source).

We now get to the point where we determine if the system can meet our need. This is where the Potential Acoustic Gain equation comes into play. The simplest form of this equation looks like this:

$$\mathbf{PAG = 20 \log D_1 - 20 \log D_2 + 20 \log D_0 - 20 \log D_S}$$

This form of the equation will allow us to determine the amount of gain available from the system just before the undesirable oscillation we all know as feedback occurs. It also assumes the system uses omnidirectional microphones and loudspeakers and it neglects the effects of reverberation and echo (as if the system were outdoors). I'll mention more about this later, but for now this version of the equation will be easier to work with.

Looking at the system distances as they occur in the equation we can see some obvious facts.

PAG = 20 log D_1 - 20 log D_2 + 20 log D_0 - 20 log D_S

If we want the PAG on the left side of the equation to get larger, we have to make the positive terms (D_1 and D_0) as large as possible and the negative terms (D_2 and D_S) as small as possible on the right side of the equation.

Notice that if the loudspeaker is moved farther from the microphone (as D_1 increases), the potential gain before feedback is increases. Unfortunately, with D_0 there is little change possible since it is usually fixed by the layout of the room.

When the loudspeaker is moved closer to the listener, decreasing the negative term D_2 on the right side of the equation, the system gain increases on the left side. Finally, concerning the most important distance to change as we'll see later, when the talker moves closer to the microphone it decreases D_S and increases PAG.

THEORY MEETS REALITY IN A SOUND SYSTEM

As I mentioned earlier, there are complications when we enter the real world. The variations in direct sound compared to reflected sound, the echoes, the presence of people, the atmosphere, and more conspire to make each position in a room acoustically different. This affects feedback, as anyone who has tried to set up a sound system in a gym will tell you.

The most important point to remember as we work through these equations is this: ***if the dimensions of a sound system don't provide the needed gain with the theoretical free field conditions, then they are even less likely to work under real world conditions.***

To use these ideas in the equation we need the complete picture of our prospective sound system layout—the specific distances D_1, D_0, D_2, and D_S. Using the conference table we've already looked at, let's say that the room dimensions are 20' by 30' with a 10' ceiling. A possible set of system dimensions might be something like this (see Figure 2):

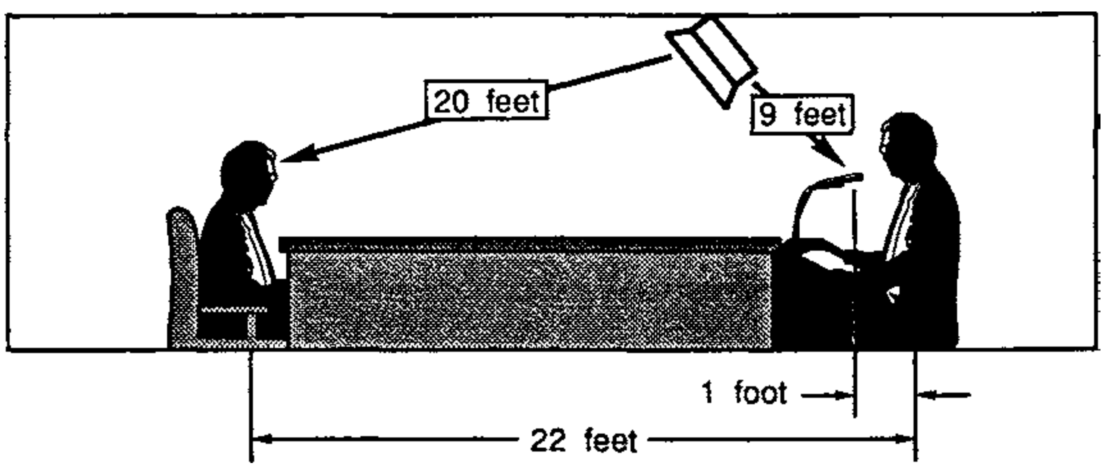

Figure 2. The dimensions of a possible sound system.

D_1 (mic to closest loudspeaker) = 9'   **D_2 (loudspeaker to farthest listener) = 20'**

D_0 (talker to listener) = 22'   **D_S (talker to mic) = 1'**

Now we simply plug these values into the equation and do the logarithms and arithmetic the same way we did with the inverse square law equation.

PAG = 20 X log of 9 - 20 X log of 20 + 20 X log of 22 - 20 X log of 1

Working backwards as before, enter 1 and hit "log." Since the logarithm of 1 is zero this term will drop out. Now enter 22, hit "log", and multiply by 20 to get 27. Save it. and do the same with 20 and 9 and save each of the answers. We wind up with 19 minus 26 plus 27 for a potential acoustic gain in the system of 20dB.

PAG = 20 X log of 9 - 20 X log of 20 + 20 X log of 22 - 20 X log of 1
= 19 - 26 + 27 - 0

PAG = 20dB

That seems like a lot, but not enough to help the listener at the far end of the table who needs 21dB of gain to be able to hear the talker as well as the listener close to the talker does.

So let's change the dimensions of the system to bring the loudspeaker closer to the listener and farther from the microphone (see Figure 3):

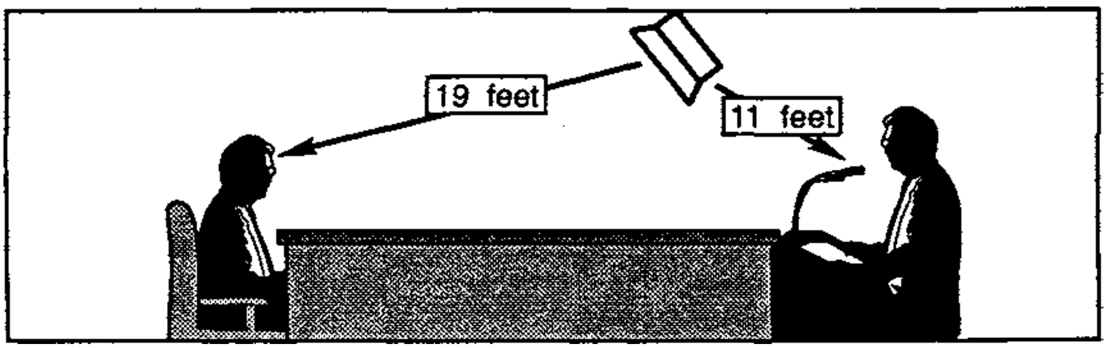

Figure 3. Revised system dimensions: loudspeaker closer to listener and farther from the microphone.

D_1=11' D_2=19' D_0=22' D_S=1'

This time after we do the logs we'll get:

PAG = 20 X 1.04139 - 20 X 1.27875 + 20 X 1.34242 - 20 X 0

Do the arithmetic and we get:

PAG = 22dB

This is enough gain so that the listener at the far end of the table hears as well as the listener near the talker without the system going into feedback. The potential gain minus the needed gain leaves a margin of 1dB which seems like what we needed when we started to look at this conference room. But there's a catch.

THE FEEDBACK STABILITY MARGIN

The version of the equation that we've been using gives us the potential gain of the system at the point *just before it starts to feedback*, the point of *unity gain.* Virtually all systems need to be operated with a safety margin (called the feedback stability margin), usually 6dB, to avoid the annoying ringing sound associated with a pre-feedback condition. When this margin is included, the equation looks like this:

$$\textbf{PAG} = 20 \log D_1 - 20 \log D_2 + 20 \log D_0 - 20 \log D_S - 6$$

Unfortunately this little minus 6 means that the gain from our latest version of the system is now reduced to 16dB — not enough to meet the 21dB needed. So we're back to the drawing board.

Let's try to move the loudspeaker even closer to the listener and further from the microphone (see Figure 4):

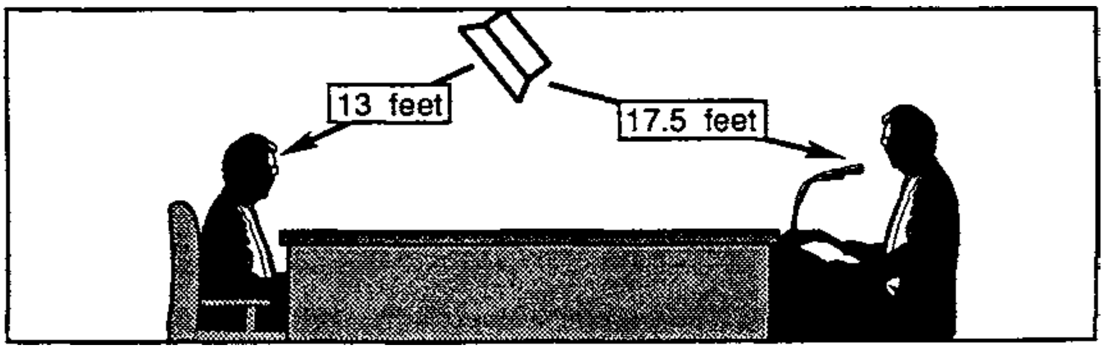

Figure 4. Loudspeaker moved even closer to the listener.

D_1=17.5' D_2=13' D_0=22' D_S=1'

The result looks good this time even with the feedback margin:

PAG = 25 - 22 + 27 - 0 - 6

PAG = 24dB

We're there…and with a 3db margin to spare. If we operate the system just as described, the farthest listener will hear as well as the nearest without the system feeding back. Our conference table system works…almost.

THE NUMBER OF OPEN MICROPHONES— THE LAST HURDLE

All along we've assumed that this conference room uses only one microphone at a time. If more microphones are open—for example, if microphones are placed and turned on in from of multiple talkers at the table—another term is added to the equation which can affect the acoustic gain of the system. Multiple open microphones create a greater risk of feedback in the system and degrade the quality of the sound due to comb filtering and increased reverberation.

The new term for the Number of Open Microphones (NOM) looks like this in our final version of the equation:

$$\textbf{PAG} = 20 \log D_1 - 20 \log D_2 + 20 \log D_0 - 20 \log D_S - 10 \log \text{NOM} - 6$$

Since this term is another logarithm term and the logarithm of 1 (for a single microphone) is 0, a system with only one microphone is not affected by including it in the calculation. But if we change from 1 microphone turned on to 2, the NOM term turns out to be 3dB (10 times the log of 2). Looking at our latest version of the system (where we had only 3dB to spare when we subtracted needed acoustic gain from the potential acoustic gain) this extra open microphone would affect our margin. We would have to subtract another 3dB from our latest 24dB PAG. If we then subtract the 21dB NAG from the new 11dB PAG, the 0dB result means the system will start to use up the feedback margin just as it provides the needed gain to the farthest listener.

Now if we try to change from 2 to 4 open microphones, the problem is further compounded from 3dB to 6dB. In fact, each time the NOW doubles, the PAG is decreased by 3dB. If we get t o8 open microphones, we've eliminated any safety margin and are back on the verge of feedback.

This essentially means that ***the number of open microphones needs to be limited as much as possible.*** Most ways of doing this have practical disadvantages: having a sound person turn off the microphones that are not being used during the conference (expensive, and difficult with only two hands); having a switch on each microphone for the user (users must be taught and often forget to use it); or using only one microphone and passing it around (very impractical logistically).

THE AUTOMATIC MICROPHONE MIXER SOLUTION

The most practical and effective solution is an automatic microphone mixer. You need to be careful here, though, because not all automatic mixers limit the number of open microphones equally well. Most systems turn on microphones when the sound they pick up is louder than a particular reference level called a "threshold." They will then turn off when the sound level drops below the threshold. This threshold may be a fixed level, and adjustable level, or an automatically adjustable level.

A weakness exists in these systems when they try to distinguish between the desired sound (such as a talker) and undesired sound (random background noise). If the background sound is loud enough, it will activate the microphone unless the threshold is set to a higher level. Then, if the background noise drops, the normal talker's level may not activate the microphone unless the threshold level is lowered as well.

This problem is addressed in certain other systems with integrated microphones which can actually sense the location of the desired sound source relative to the background noise. The microphones are then activated only when the sound comes from the desired direction, which eliminates the need for any threshold adjustments.

Another new automatic mixer technology solves the problem differently and still avoids the threshold adjustment problem. Mixers using this technology can sense the difference between irregular sound (like speech) and regular sound (like air conditioning). The mixer only turns on a channel when the signal level is louder than the background sound. This system also chooses only one microphone for each talker even if multiple microphones are "hearing" him.

THE QUICKEST FIX

It may have occurred to you that the only factors that we've tried to alter in the system have been D_1 and D_2. Of course it's not very likely that you'll be able to adjust the talker-to-listener distance, D_0, in most real systems, but an easy way to improve the acoustic gain of a system is by adjusting the remaining distance, D_S.

Remember in the inverse square law free field calculations, where we doubled the distance of the listener from the sound source, that we calculated a 6dB drop in the sound level as perceived by that listener. This also must work in reverse: if the distance changes from 8 feet to 4 feet, the perceived level will *increase* by 6dB.

As I mentioned earlier, D_S is where the doubling effect of the inverse square law really shows some dramatic changes in the system gain. This is because of the relatively short distances involved in doubling. When the talker doubles her distance from the microphone, the system gain drops by 6dB. If the change is instead from 1 foot to ½ foot (.5), then the system gain will increase by 6dB. Try these calculations yourself in the PAG equation by changing only D_S and leaving the other distances the same as in the last version of the system we used. First change it from 1 foot to 2 feet. Then try changing it from 1 foot to .5 foot.

This is sometimes an easy solution and is actually one of the first rules in microphone application: ***get the microphone as close as possible to the sound source.*** Compared to moving loudspeakers to change D_1 and D_2, it is also often the easiest distance in the system to change. This is simply because it is generally a much shorter distance to begin with and therefore much easier to cut in half to get that extra 6dB. The concept of critical distance also plays a part here. Since it's generally true that when D_S is greater than the critical distance of the talker, the result is more reverberance and a loss of intelligibility.

In the other parts of the system—D_1 and D_2—the receiver (the listener or microphone) is generally outside the critical distance of the source (the talker or loudspeaker). When this is the case for D_1, the signal received by the microphone is not as low as predicted by the inverse square law, resulting in less potential gain than expected for the system. For D_2, this higher-than-expected level at the receiver actually gives the listener somewhat more sound than predicted. Here, however, working with loudspeaker placement and echoes is important since the added level is in part due to reflected sound. Early echoes tend to help intelligibility and later ones hurt it. So if a loudspeaker needs to be "somewhat near" the ceiling, put it close to the ceiling to take advantage of the early echoes produced within a few feet of the reflective surface.

Without delving too far into the concept of speaker placement, let me just say that multiple speaker locations tend to act like late echoes if they arc far apart. The resulting comb filtering and loss of intelligibility is minimized by a single loudspeaker or closely spaced array if it can be placed far from the talker and close enough to the listeners.

THE WORST CASE

Finally, with the multiple microphone problem under our belts we need to deal with the fact that, unlike our theoretical system, most systems include multiple loudspeakers. The most practical approach in determining the distances, including specific loudspeaker distances to use in the PAG equation, is to consider the worst case for each. Therefore, as I stated earlier, you would use the most distant listener for Do. You should also pick the largest expected distance for the microphone to talker for D_S; the loudspeaker closest to the farthest listener for D_2; and the loudspeaker *closest* to the microphone for D_1. These choices will give the most accurate representation of the acoustic gain in the system.

A LITTLE KNOWLEDGE

I mentioned earlier that the microphones in the theoretical system we calculated were omnidirectional. Because unidirectional microphones have the ability to target desired sounds (the talker in our example) and reject unwanted sounds (the output from the loudspeaker), they can provide a margin of extra gain in a system. So can directional loudspeakers. But they are not a magic bullet. The practical limit on the improvement that these components can make is usually considered to be about 6dB. Although theory and component specifications seem to indicate a 6dB improvement for *each* component, real world conditions provide severe performance limitations.

The second assumption was that the system was not affected by reverberation and echo. Virtually any indoor system *will* be affected by these conditions and the acoustic gain and performance of the system will be limited by them. This article will not explore the vast topic of room acoustics, but here's something to keep in mind as you think about the PAG equation. If the system won't provide enough gain when you ignore these factors (as we have calculated in this article) it almost certainly won't work when they're included.

SUMMARY

In working through the math or just reading through this information, the major points to remember are:

1. To make a significant change in the gain of a sound system before it feeds back, distances need to be doubled or cut in half. (Inverse Square Law)
2. Changes to improve the potential acoustic gain of a system involve:
   a. Making the loudspeaker-to-microphone distance, D_1, as large as possible.
   b. Making the loudspeaker-to-listener distance, D_2, as small as possible.
   c. Most importantly and easiest, making the talker-to-microphone distance, D_S, as small as possible.
3. Limiting the number of open microphones will also improve the potential acoustic gain of the system.

If you're interested in studying these topics further, the following books are suggested, many of which were drawn from when writing this article.

• *Handbook of Sound System Design*, John Eargle;
ELAR Publishing Co., Inc., Commack, NY 11725

• *Handbook for Sound Engineers,* Ed. Glen Ballou
(especially the article by Chris Foreman "Sound System Design")
Howard W. Sams & Company, A Division of Macmillian, Inc., 4300
West 62nd Street. Indianapolis. Indiana 46268

• *Architectural Acoustics,* M. David Egan; McGraw-Hill Book Company

• *The Gain of a Sound System,* by C.P. Boner and R.E. Boner (the original article on the topic)

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