A Critical Feedback Analysis

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The author presents a simple and easily understandable concept of the effect of feedback on input and output impedances of tubes around which a feedback path is provided.

One of the earliest applications of positive feedback occurred when regenerative detectors made possible the reception of weak signals with only one- or two-tube sets. Even today, some receivers still use regenerative intermediate frequency stages to obtain greater pulling power.

The use of negative feedback began in the earliest days of the vacuum tube when neutralization was employed to stabilize radio frequency amplifiers in both receivers and transmitters. The development of multigrid tubes eliminates the necessity of neutralization, although all triode transmitter stages today still incorporate one of several methods of neutralization—circuits in which varying magnitudes of out-of-phase plate voltages are coupled back to grid circuits to obtain cancellation of grid-plate interaction. The entire field of electronics today demands greater stage stability, wider band response, and lower noise and distortion levels. Along with other precautions, all of these requirements can be approximated by the judicious application of negative feedback.

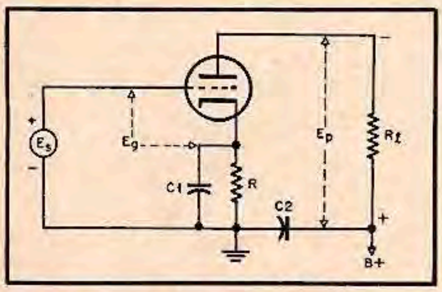

Fig. 1. Basic amplifier circuit without feedback.

A sizeable amount of literature has appeared during the last few years on the impedance changes that take place in an amplifier with feedback, notably that associated with the cathode-follower. A study of this material discloses a marked divergence of opinion among several members of the profession on various principles that underlie the operation of a feedback amplifier, among which are the following:

1. The manner in which gain reduction takes place.
2. Impedance changes, if any, of tubes within a feedback loop.
3. An exact mathematical procedure for determining the improvement attributed to feedback.

It is the intent of this paper to outline the effects of energy interchange between amplifier grid and plate circuits. The points wherein this analysis differs from that published in some papers of the past are as follows:

1. The mathematical sign of β is never negative.
2. Feedback affects only the input, not the output impedance.
3. Feedback, either negative or positive, does not affect the plate resistance of any tubes in a feedback loop.

To review briefly an amplifier without feedback, *Fig.* 1 is a circuit of a 6J5 triode which has a plate resistance of 7,700 ohms and a mu of 20. The gain of any stage is expressed by the familiar

$$A = \frac{\mu R_l}{R_p + R_l} \qquad (1)$$

where μ = amplification factor of the tube

R_p = plate resistance of the tube

R_l = plate circuit load

If the plate load for the 6J5 is made equal to the plate resistance, the gain from Eq. (*1*) will be

$$A = \frac{-20(7{,}700)}{7{,}700 + 7{,}700} \qquad (2)$$

$$= \frac{-154{,}000}{15{,}400}$$

$$= -10.0, \text{ equivalent to 20.0 decibels.}$$

In any plate-loaded amplifier calculations, the phase turnover must be accounted for; the amplifier gain A must appear in all work with a negative sign since the output is 180 deg. out of phase with the input. For all audio and r.f. applications where the electron transit time does not enter into calculations, the mu may be written as $\mu\underline{/180°}$. In Eq. (*2*), the mu is understood to be $20\underline{/180°}$, or simply $-20 + J0$. A plate current change through R_l will produce an output voltage

$$E_p = AE_g \qquad (3)$$

where E_g = the grid-to-cathode input voltage.

Referring to *Fig.* 1, the grid circuit terminology includes E_s and E_g, denoting source and sink voltages respectively.

Figure 3 shows the 7,700-ohm load line that the 6J5 of *Fig.* 1 is working under; a plate supply of 275 volts will drive a 6.0-ma current through the load and also produce a bias of 8.0 volts across a 1333-ohm bias resistor. A 5.0-volt signal E_s will produce a 5.0-volt drop, E_g, across the input resistance R_g of the tube so the output voltage from Eq. (*3*) will be

$$E_p = -10(5.0) = -50.0 \text{ volts.} \qquad (4)$$

Introducing Feedback

To make the amplifier of *Fig.* 1 degenerative, the load resistor is divided into two parts, R_o and R_c, the grid return being made to the junction of these two resistors, thereby making the voltage E_c, common to both grid and plate circuits. The amount of feedback in this stage will be governed by the amplitude of the feedback voltage E_c which, in turn, is determined by the ratio R_c/R_l also known as β, since β itself can be defined as the percentage of the total output voltage that is fed back for feedback purposes. Mathematically this is

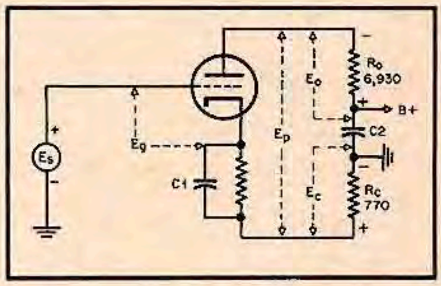

Fig. 2. Rearrangement of circuit elements to provide feedback and to clarify operation of circuit.

$$K = \frac{R_c}{R_l} = \frac{E_c}{E_p} = \frac{E_c}{AE_g} \qquad (5)$$

from which the resistors R_o and R_c are computed as

$$R_c = KR_l \qquad (6)$$

and

$$R_o = R_l - R_c \qquad (7)$$

The values of β can be chosen between the limits of unity and zero since it is always an expression of percentage. In view of this fact, the sign of β is never negative since none of the quantities in Eqs. (*5*), (*6*), or (*7*) are negative in sign. A *de*generative amplifier has its feedback voltage arranged to oppose the input signal, whereas, a *re*generative amplifier allows the feedback voltage to assist the signal by being in phase with it.

To make an analysis of the *Fig.* 2 amplifier, let

$$K = 0.10, \text{ (10 per cent of output voltage fed back)} \qquad (8)$$

To assist in comparision, calculations with feedback will retain the same 7,700-ohm total plate load to keep $A = -10.0$. The bias resistor from Eq. (*6*) will be

$$R_c = 0.10(7{,}700) = 770 \text{ ohms} \qquad (9)$$

from which by Eq. (*7*)

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$$R_o = 7{,}700 - 770$$
$$= 6{,}930 \text{ ohms.} \qquad (10)$$

To obtain the proper 8.0-volt bias, a by-passed 560-ohm resistor must be added in series with R_c. The plate circuit load is still 7,700-ohms. The plate circuit voltage gains and losses can be summed up as

$$E_p = E_o + E_c \qquad (11a)$$

or

$$E_o = E_p - E_c \qquad (11b)$$

The output voltage E_o to ground can be found by substituting in Eq. (*11b*) for E_p and E_c from Eqs. (*2*) and (*5*) so that

$$E_o = AE_g - KAE_g$$
$$= AE_g(1-K) \qquad (12)$$

Substituting circuit values from *Fig.* 2 with a grid drive E_g equal to 5.0 volts, as in *Fig.* 1, and a total plate circuit voltage of −50.0 volts, the output voltage available from plate to ground is

$$E_o = -10.0(5.0)(1.00-0.10) \qquad (13)$$
$$= -50.0(0.90)$$
$$= -45.0 \text{ volts.}$$

Five of the fifty volts generated in the plate circuit exist across the bias resistor R_c as a feedback voltage; since the plate circuit is a voltage divider, the output voltage E_o is 0.9151 db lower than the amount generated in this circuit.

Effect on Input Signal

In addition to the plate-circuit loss, the feedback voltage E_c also acts to change the voltage equation in the grid circuit. These grid-circuit voltages can be stated as

$$E_s = E_g - E_c \qquad (14)$$

Substituting in this equation from *Fig. 2*, the signal E_s with feedback

$$E_s = 5.0-(-5.0)$$
$$= 10.0 \text{ volts.} \qquad (15)$$

The grid voltage E_g was assumed to be 5.0 volts; with a β factor of 0.10, there will be −5.0 volts of feedback across R_c. Furthermore, only one-half of the input signal E_s reaches the grid-cathode terminals, the other half being used to overcome the out-of-phase feedback voltage E_c. The loss of gain between E_s and E_g amounts to 6.0206 decibels which added to the plate circuit loss of 0.9151 db totals 6.9357 db. The gain with feedback is equal to

$$20 \log \left(\frac{E_o}{E_s}\right) = 13.0642 \text{ db.} \qquad (16)$$

The 6.9357 db loss due to feedback plus the gain above is equal to 19.9999 db, the gain without feedback determined in Eq. (*3*). Computations in this paper are carried out to a sufficient number of places to indicate the accuracy of the theory; problems in feedback usually include small differences of potential, so to obtain answers to three or four place accuracy, six- or seven-place figures are often necessary in the work; however, this paper does not imply that seven or even three place accuracy is necessary to solve any or all feedback problems.

An accurate equation of over-all stage gain in amplifiers using an unbypassed

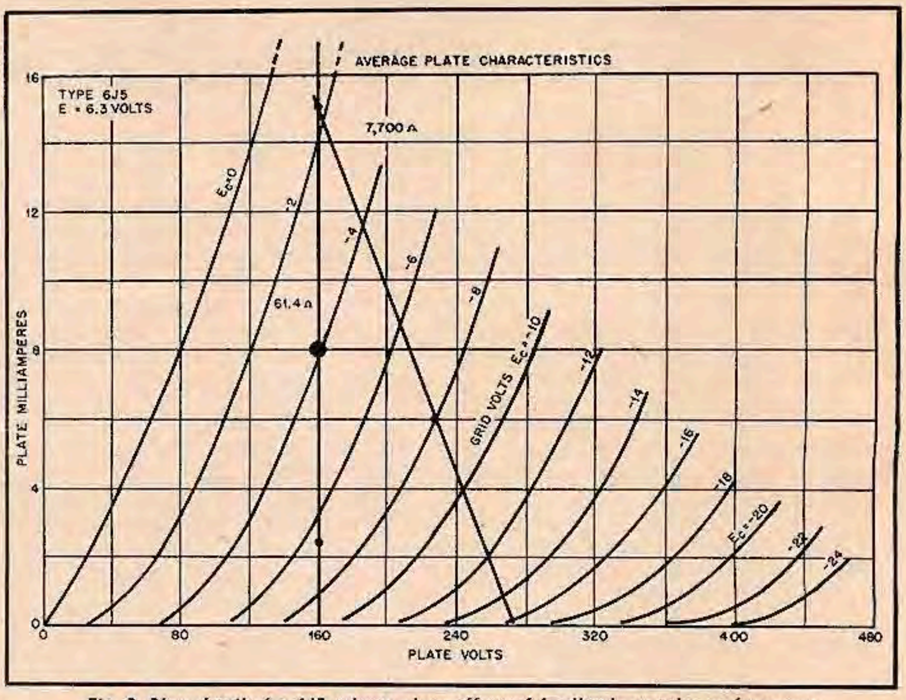

Fig. 3. Plate family for 6J5 tube to show effect of feedback on tube performance.

bias resistor to develop the feedback voltage can be obtained by using Eqs. (*12*) and (*5*) in equation

$$E_s = E_g - E_c \qquad (14)$$
$$= \frac{E_o}{A(1-K)} - KAE_g$$

and again for E_g

$$= \frac{E_o}{A(1-K)} - \frac{KAE_o}{A(1-K)}$$
$$= \frac{E_o(1-AK)}{A(1-K)}$$

and rearranging for over-all gain

$$\frac{E_o}{E_s} = \frac{A(1-K)}{(1-AK)} \qquad (17)$$

In this equation, both of the factors that account for the loss of gain in this amplifier are included—(*1–K*) accounts for the plate-circuit loss and (*1–AK*) for the grid-circuit attenuation. This change of amplification has not been assigned to a fictitious or apparent change of tube mu or plate resistance since both these tube parameters have been carried through in all calculations as stated in tube manuals.

By using equations developed in this paper, the *Fig.* 2 amplifier can be modified to have unity gain by changing the β value to 0.45, in which case the output voltage using Eq. (*13*) will be

$$E_o = AE_g(1-K) \qquad (18)$$
$$= -10.0(5.0)(1.00-0.45)$$
$$= -50.0(0.55)$$
$$= -27.5 \text{ volts.}$$

The gain A and E_g will remain unchanged since the total plate load is still 7,700 ohms. The input signal E_s to obtain the above output is computed by rearranging Eq. (*17*) so that

$$E_s = \frac{E_o(1-AK)}{A(1-K)} \qquad (19)$$
$$= \frac{[-27.5(1.00) - (-10.0)(0.45)]}{-10.0(1.00-0.45)}$$
$$= \frac{-151.25}{-5.50}$$
$$= 27.5 \text{ volts.}$$

Effect of Increasing Feedback

The feedback voltage developed across R_c will be the difference between the total output E_p of −50.0 volts and E_o from Eq. (*18*) equal to −27.5 volts or −22.5 volts. When β is increased to 0.50, the voltages E_o and E_c will be equal in amplitude, a condition necessary for phase-inverter service. Cathode-follower operation results when β is made equal to unity, in which case the entire plate load is R_c, since R_o is zero in this case. Notably in television work, the load impedances often used with cathode followers are much lower than plate-loaded applications. In these applications, the gain A is reduced considerably, thereby producing smaller feedback voltages across the load. As long as load impedances are fairly high, normal plate-voltage values can be used since the plate current will be limited by the IR drop in the load. With loads in the neighborhood of 75 to 300 ohms, plate voltages must be reduced considerably; this is confirmed by reference to the 6J5 curve in *Fig.* 3. A low value of load will cause the load line to approach the vertical as the load impedance approaches zero. The operating point must be chosen so the tube will not be driven into either cutoff or saturation regions. The 6J5 is shown operating with 160 plate volts, which will cause a plate dissipation of 1.28 watts, one-half of its rated 2.5 watts. A bias of 4.0 volts places the operation about midway on the graph at this point; thus the 8-ma plotted current will necessitate a load resistor of 500 ohms to produce the proper bias. Assuming the tube is being used to feed a properly terminated 70-ohm transmission line, the tube as a generator will actually look into a

61.4-ohm load, which is the parallel combination of 500 and 70 ohms. The amplifier gain A with this load from Eq. (*1*) will be

$$A = \frac{-20(61.4)}{7{,}700 + 61.4} \qquad (20)$$
$$= -0.1582$$

This tube, even if it were plate-loaded, would be an attenuation amplifier since its load is so small that it operates at less than unity gain. An equation for output voltage can be obtained by rearranging several previously developed equations for the voltage E_c when all the load is made up of the bias resistor. Using Eq. (*5*) with substitutions from Eqs. (*7*) and (*12*), we have

$$E_s = \frac{AE_g(1-K)(1-AK)}{A(1-K)} \qquad (21)$$
$$= E_g(1-AK) \text{ or } E_g = \frac{E_s}{(1-AK)}$$

which substituted in Eq. (*5*) for E_g

$$E_c = \frac{KAE_s}{(1-AK)} \qquad (22)$$

This is an accurate statement of the voltage developed across an unbypassed bias resistor in a cathode-follower stage when the input E_s, the gain A, and the β factor are all known.

From *Fig*. 3, a grid drive of approximately 2.5 volts is indicated since anything greater will drive the tube into nonlinear operation. A 2.5-volt drive will swing the plate current between 2.5 and 15 milliamperes. The output E_c will be

$$E_c = \frac{(1.0)(-0.1582)(2.5)}{[1.00-(-0.1582)(1.0)]} \qquad (23)$$
$$= \frac{-0.39550}{1.1582}$$
$$= -0.34148 \text{ volts.}$$

This voltage is in series opposition to the signal E_s, so the actual voltage E_g will be *2.50000 – 0.34148* or *2.15852* volts. To check the accuracy of the previous equations, the voltage E_g times the gain A should produce the output voltage E_c. Carrying this out to seven places, the answer is – 0.3414778 volts.

The ability of a cathode-follower to counteract changes in the load it faces can be demonstrated by assuming the previous 70-ohm load changed to 700 ohms which is equal to a 20-db change. The new gain A from Eq. (*1*)

$$A = \frac{-20(291.6)}{7700 + 291.6} \qquad (24)$$
$$= \frac{-5832}{7991.6}$$
$$A = -0.7297$$

where the parallel combination of the 500-ohm load resistor and the 700-ohm line is equal to 291.6 ohms.

Assuming the same 2.5-volt input signal, the output by Eq. (*22*) will be

$$E_c = \frac{-0.7297(1.00)(2.5)}{[1.00-(-0.7297)(1.0)]} \qquad (25)$$
$$= \frac{-1.82425}{1.7297}$$
$$= -1.0546 \text{ volts.}$$

This amounts to an increase of 3.09 times that of the 70-ohm load, equivalent to 9.90 db. The 20-db change in line impedance is not all reflected to the tube since it is in parallel with R_c. Actually, the tube looks into a variation of only 13.56 db. The amount of feedback under any feedback conditions is equal to (*1–AK*), the change in gain that occurs in the grid circuit. For negative feedback, the gain is always negative; positive feedback, or regeneration, permits a higher voltage to reach the grid than that which is fed in. This same factor, (*1–AK*), is also the amount by which noise and distortion are attenuated in a negative feedback amplifier. Under the 700-ohm load, the grid-cathode voltage E_g will be $E_s + E_c = 2.5000 - 1.0546 = 1.4454$ volts. The feedback will be

$$20 \log \frac{E_s}{E_g} = 20 \log \left(\frac{2.5000}{1.4450}\right)$$
$$= 20(0.239) = 4.78 \text{ db.} \qquad (26)$$

The amount of feedback at the 70-ohm load will 1.28 db, nearly one-fourth that with the 700-ohm load. Larger amounts of feedback result by using beam pentodes such as the 6AG7 and 6AH6 will give a gain of four. With driving signals of 3.0 volts, the output E_c at video line impedances of 70 ohms will be approximately 2.4 volts. Between 10 and 12 db of feedback are the advantages gained by using higher conductance pentodes.

The use of voltage feedback in an amplifier, diagrammed in *Fig*. 4, is accomplished by bridging two resistors,

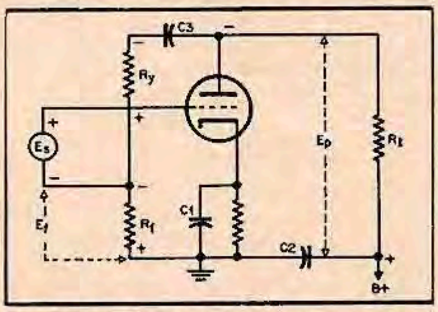

Fig. 4. Method of introducing voltage feedback in series with source of signal.

R_y and R_f, across the normal plate load R_t. There are several other variations possible when using voltage feedback—using R_y and R_f themselves as a load, or the simple expedient shown in *Fig*. 5.

In *Fig*. 4, resistors are proportioned so

$$K = \frac{R_f}{R_f + R_y} \qquad (27)$$

Assuming the total plate load is still 7,700 ohms, the gain A will also be – 10.0. Assuming R_y and R_f in series at 50,000 ohms, R_t will have to be increased to 9,102 ohms so the combination will equal 7,700 ohms. The β factor, K, will then be 0.10 if R_y is 45,000 and R_f is 5000. There is no division of the plate-circuit output voltage E_p as was the case with *Fig*. 2: there is no (1–K) factor. The output E_p will be – 50.0 volts if the grid E_g receives 5.0 volts. The input must, however, be 10.0 volts to overcome the – 5.0 volts of feedback across R_f. The over-all gain of this stage is

$$20 \log\left(\frac{50.0}{10.0}\right) = 13.9794 \text{ db.} \qquad (28)$$

This is a difference of 0.9151 db over *Fig*. 2, accounting for the (*1–K*) plate-circuit factor.

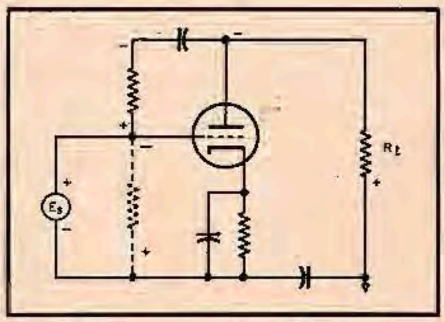

Fig. 5. Rearrangement of Fig. 4 to show introduction of feedback in parallel with source.

Reduction of Noise and Distortion

The remainder of this paper will be devoted to an explanation of how both current and voltage feedback reduce noise and distortion. *Figure* 6 shows two feedback paths around the 6J5 tube using $E_g = 5.0$ volts. $R_t = 7{,}700$ ohms and $E_p = -50.0$ volts. The grid circuit equation will be

$$E_s + E_g + E_f + E_c = 0.00 \qquad (29)$$

Identifying K_1 with the current-feedback circuit and K_2 with plate-voltage feedback, the following are true statements:

$$E_c = K_1 E_p \qquad (30)$$

and

$$E_f = K_2 E_p \qquad (31)$$

Assuming $K_1 = 0.100$ and $K_2 = 0.200$, the above equation with substitutions are:

$$E_c = (0.100)(-50.0) \qquad (32)$$
$$= -5.00 \text{ volts}$$

and

$$E_f = (0.200)(-50.0) \qquad (33)$$
$$= -10.0 \text{ volts.}$$

The input signal E_s by Eq. (*29*) with this substitution will be

$$E_s = -(-5.0) - (-10.0) - (-5.0) \qquad (34)$$
$$= 20.0 \text{ volts.}$$

Since there are – 15.0 volts of feedback potential opposing E_s, the input must

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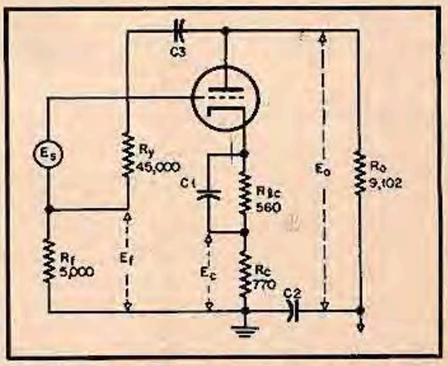

Fig. 6. Typical circuit involving voltage feedback in series with source and current feedback due to unbypassed cathode resistor.

FEEDBACK

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be 20.0 volts as indicated so that the drive E_g will have a net value of 5.0 volts.

The plate circuit has a loss of 5.0 volts; therefore, the output E_o will be −45.0 volts as in *Fig.* 2. The over-all gain is

$$20 \log \left(\frac{45.0}{20.0}\right) = 7.0436 \text{ db.} \qquad (35)$$

The loss due to feedback is equal to the gain without feedback, 20.0 db, minus the gain from *(35)* or 12.9564 db. This is not the amount by which noise and distortion are reduced, however, since the additional plate loss of 0.9151 db does not contribute to this cause. The feedback factor for the two paths is

$$1 - A_1K_1 - A_2K_2,$$

and substituting we have

$$1.00-(-10.0)(0.10)-(-10.0)(0.20) =$$
$$1.00-(-1.00)-(-2.00) = 4.00,$$

equivalent to 12.0413 db.

For purposes of computation, assume that this amplifier without feedback would generate 10 per cent of second harmonics and has a noise level of −40 db. The noise and distortion plate-circuit voltages without feedback will be (−50.0)(0.10) = −5.0 volts distortion and (0.01)(−50.0) = −0.50 volts noise since −40 db is equivalent to .01. Feedback will improve the noise by 12.04 db while the distortion will be reduced by the factor 4.0. The noise voltage, also reduced by one-fourth, is −0.125 volts with feedback. The two β circuits will feed back 0.30 of this voltage or −0.0375 volts. This noise voltage has nothing to cancel it in the grid circuit so is re-amplified by the gain −10.0 and produces

$$nE_p = A(nE_c + nE_f) \qquad (36)$$
$$= -10.0(-0.0375)$$
$$= 0.375 \text{ volts.}$$

This voltage is out of phase with the original noise voltage of −0.500 volts so will cancel all but −0.125 volts. The new hum level is

$$20 \log \left(\frac{-50.0}{-0.125}\right) = \qquad (37)$$
$$20(2.60206) = -52.0402 \text{ db}$$

which checks the theory since the original noise plus the improvement equals the above value.

The distortion voltage reduction follows the same pattern; the voltage fed back will be (−1.25)(0.30) = −0.375 volts, which when amplified is

$$dE_p = A(dE_c + dE_f) \qquad (38)$$
$$= -10.0(-0.375)$$
$$= 3.75 \text{ volts.}$$

This feedback voltage added to the distortion being generated continuously will reduce the modulation by-products by the amount of "gain" change in the input circuit.

The author hopes that this paper will aid other engineers to obtain a clearer conception of feedback principles, thereby expanding the field of uses to which both positive and negative feedback may be applied.