THE TEMPERED MUSICAL SCALE

Computer print-out to eleven significant figures of octaves based on A—448Hz to A—425Hz.

The figures set out below are probably the most detailed and accurate of the type ever published. They relate to the octave above middle C and are for the tempered scale, where the semitone intervals have a relationship based on the twelfth root of 2.

The frequencies throughout the octave are shown for each value of A and, alongside them the reciprocals, which may be the preferred reading figure when using a digital frequency meter with a large number of available display digits. Quantitatively, the reciprocal represents the duration of 1 cycle of the indicated frequency in seconds to the power —3 (shown by the expression E-03); in other words, it is the duration of one cycle of the indicated frequency in milliseconds.

The print-out was made available to "Electronics Australia" by Mr Eric Mirovitch, who is currently doing development work on a tuning instrument based on a logic type divider fed from a master oscillator and switched to produce the required semitone intervals.

The program was run at the Sydney branch of the Division of Computer Research, C.S.I.R.O., by courtesy of Mr C. H. Gray, officer in charge, and Miss Joan Hayhurst.

To avoid risk of error, the figures were not re-set for reproduction but were photographed directly from the original computer print-out and re-touched by hand where necessary. The slight misalignment and the variations in line thickness are therefore largely a heritage of the original high-speed print-out — but they are also an indication of authenticity.

In terms of frequency, the ultimate accuracy represented by the figures is far in excess of normal musical requirements which seldom exceed 0.1 per cent. The print-out can be used, however, to put in order earlier tables which may contain errors or which may have been rendered ambiguous by "rounding off." More particularly, the figures will be of use to those fortunate enough to have access to a digital frequency meter for tuning. They may also assist in designing tuning instruments relying on beats, synthesis or division from a common source.

In an idealised situation, the only octave of interest would be that based on A—440Hz. Using these figures and a digital frequency meter, it would be a simple matter to tune an electronic organ to standard pitch.

A pipe organ, however, presents a quite different problem because of the complexity of the tuning procedure, the nature of the adjustments involved, and the day-to-day effect of temperature and humidity variations. These all provide a strong incentive to tune such an instrument to itself and minimise the number and the extent of adjustments to the pipes on any single occasion. If observation suggests that this will be most easily achieved by tuning on the basis of A—438Hz, then this is what most likely will be done.

The figures should cover the pitch settings which might conceivably be encountered in practical — though sometimes venerable — instruments. Rarely, if ever, should anyone be involved in a pitch above A-448 or below A-425.

(An article on tuning was presented in the June, 1965 issue of "Electronics Australia." Written by the Editor, Neville Williams, it was entitled: "Musical Scales and Tuning — in matter-of-fact electronic terms.")

A= 446

530.38637325	C	1.8854179715E-03
500.61807352	B	1.9975307583E-03
472.52054007	A+	2.1163101182E-03
* 446.00000000	A	2.2421524663E-03
420.96794346	G+	2.3754777900E-03
397.34082831	G	2.5167310499E-03
375.03980123	F+	2.6663836658E-03
353.99043463	F	2.8249350891E-03
334.12247819	E	2.9929144708E-03
315.36962446	D+	3.1708824263E-03
297.66928753	D	3.3594329071E-03
280.96239419	C+	3.5591951830E-03
265.19318671	C	3.7708359419E-03

A= 448

532.76478748	C	1.8770009271E-03
502.86299762	B	1.9886132102E-03
474.63946626	A+	2.1068623051E-03
* 448.00000000	A	2.2321428572E-03
422.85569209	G+	2.3648729784E-03
399.12262575	G	2.5054956434E-03
376.72159406	F+	2.6544801673E-03
355.57783568	F	2.8123237718E-03
335.62078527	E	2.9795532455E-03
316.78383802	D+	3.1567267012E-03
299.00412739	D	3.3444354387E-03
282.22231524	C+	3.5433059188E-03
266.38239382	C	3.7540018529E-03

A= 445

529.19716614	C	1.8896548659E-03
499.49561147	B	2.0020195914E-03
471.46107698	A+	2.1210658712E-03
* 445.00000000	A	2.2471910112E-03
420.02406915	G+	2.3808159423E-03
396.44992959	G	2.5223866254E-03
374.19890481	F+	2.6723755391E-03
353.19673410	F	2.8312832579E-03
333.37332465	E	2.9996401212E-03
314.66251767	D+	3.1780080048E-03
297.00186761	D	3.3669821947E-03
280.33243366	C+	3.5671933744E-03
264.59858315	C	3.7793097307E-03

A= 447

531.57558037	C	1.8812000342E-03
501.74053557	B	1.9930620094E-03
473.58000317	A+	2.1115756436E-03
* 447.00000000	A	2.2371364653E-03
421.91181777	G+	2.3701635220E-03
398.23172703	G	2.5111007791E-03
375.88069765	F+	2.6604186016E-03
354.78413516	F	2.8186153237E-03
334.87163173	E	2.9862189127E-03
316.07673123	D+	3.1637887297E-03
298.33670747	D	3.3519173972E-03
281.59235471	C+	3.5512327777E-03
265.78779026	C	3.7624000674E-03

A= 444

528.00795902	C	1.8939108453E-03
498.37314942	B	2.0065286446E-03
470.40161388	A+	2.1258430466E-03
* 444.00000000	A	2.2522522523E-03
419.08019484	G+	2.3861781404E-03
395.55903087	G	2.5280676762E-03
373.35800839	F+	2.6783944030E-03
352.40303358	F	2.8376600220E-03
332.62417112	E	3.0063960675E-03
313.95541090	D+	3.1851656805E-03
296.33444768	D	3.3745054878E-03
279.70247313	C+	3.5752275938E-03
264.00397959	C	3.7878216896E-03

A= 443

526.81875192	C	1.8981860391E-03	
497.25068738	B	2.0110580546E-03	
469.34215079	A+	2.1306417894E-03	
* 443.00000000	A	2.2573363431E-03	
418.13632052	G+	2.3915645471E-03	
394.66813216	G	2.5337743753E-03	
372.51711198	F+	2.6844404400E-03	
351.60933305	F	2.8440655750E-03	
331.87501758	E	3.0131825146E-03	
313.24830411	D+	3.1923556707E-03	
295.66702776	D	3.3821830171E-03	
279.07251261	C+	3.5832980849E-03	
263.40937603	C	3.7963720770E-03	

A= 442

525.62954479	C	1.9024805776E-03
496.12822533	B	2.0156079597E-03
468.28268769	A+	2.1354622459E-03
* 442.00000000	A	2.2624434389E-03
417.19244621	G+	2.3969753266E-03
393.77723344	G	2.5395068965E-03
371.67621557	F+	2.6905138346E-03
350.81563252	F	2.8505001125E-03
331.12586404	E	3.0199996696E-03
312.54119733	D+	3.1995781949E-03
294.99960782	D	3.3898350149E-03
278.44255209	C+	3.5914050941E-03
262.81477247	C	3.8049611542E-03

A= 441

524.44033767	C	1.9067945926E-03
495.00576328	B	2.0201784993E-03
467.22322460	A+	2.1403045640E-03
* 441.00000000	A	2.2675736962E-03
416.24857190	G+	2.4024106448E-03
392.88633472	G	2.5452654155E-03
370.83531915	F+	2.6966147731E-03
350.02193200	F	2.8569638316E-03
330.37671050	E	3.0268447415E-03
311.83409055	D+	3.2068334742E-03
294.33218790	D	3.3975217157E-03
277.81259156	C+	3.5995488699E-03
262.22016892	C	3.8135891839E-03

STANDARD PITCH
A=440Hz

A= 440

523.25113057	C	1.9111282166E-03
493.88330123	B	2.0247698140E-03
466.16376151	A+	2.1451688925E-03
* 440.00000000	A	2.2727272727E-03
415.30469759	G+	2.4078706689E-03
391.99543600	G	2.5510501097E-03
369.99442273	F+	2.7027434430E-03
349.22823147	F	2.8634569312E-03
329.62755696	E	3.0337269409E-03
311.12698377	D+	3.2141217322E-03
293.66476797	D	3.4052433559E-03
277.18263104	C+	3.6077296627E-03
261.62556536	C	3.8222564321E-03

A= 439

522.06192344	C	1.9154815839E-03
492.76083919	B	2.0293820460E-03
465.10429841	A+	2.1500553820E-03
* 439.00000000	A	2.2779043280E-03
414.36082327	G+	2.4133555680E-03
391.10453728	G	2.5568611578E-03
369.15352632	F+	2.7089000340E-03
348.43453094	F	2.8699796122E-03
328.87840343	E	3.0406374806E-03
310.41987699	D+	3.2214431940E-03
292.99734805	D	3.4130001744E-03
276.55267051	C+	3.6159477258E-03
261.03096180	C	3.8309631665E-03

A= 438

520.87271634	C	1.9198548295E-03
491.63837714	B	2.0340153383E-03
464.04483532	A+	2.1549641842E-03
* 438.00000000	A	2.2831050229E-03
413.41694896	G+	2.4188655122E-03
390.21363856	G	2.5626987403E-03
368.31262990	F+	2.7150847373E-03
347.64083042	F	2.8765320770E-03
328.12924989	E	3.0475795753E-03
309.71277021	D+	3.2287980870E-03
292.32992812	D	3.4207924124E-03
275.92270999	C+	3.6242033142E-03
260.43635824	C	3.8397096578E-03

A= 437

519.68350922	C	1.9242480900E-03
490.51591509	B	2.0386698357E-03
462.98537222	A+	2.1598954524E-03
* 437.00000000	A	2.2883295194E-03
412.47307464	G+	2.4244006736E-03
389.32273985	G	2.5685630395E-03
367.47173349	F+	2.7212977458E-03
346.84712990	F	2.8831145304E-03
327.38009635	E	3.0545534416E-03
309.00566342	D+	3.2361866412E-03
291.66250819	D	3.4286203126E-03
275.29274946	C+	3.6324966856E-03
259.84175469	C	3.8484961787E-03

A= 436

518.49430211	C	1.9286615030E-03
489.39345304	B	2.0433456839E-03
461.92590913	A+	2.1648493410E-03
* 436.00000000	A	2.2935779817E-03
411.52920034	G+	2.4299612255E-03
388.43184113	G	2.5744542390E-03
366.63083708	F+	2.7275392544E-03
346.05342937	F	2.8897271783E-03
326.63094281	E	3.0615592981E-03
308.29855664	D+	3.2436090876E-03
290.99508826	D	3.4364841206E-03
274.66278894	C+	3.6408281000E-03
259.24715113	C	3.8573230049E-03

A= 435

517.30509499	C	1.9330952076E-03
488.27099099	B	2.0480430303E-03
460.86644604	A+	2.1698260062E-03
* 435.00000000	A	2.2988505747E-03
410.58532602	G+	2.4355473433E-03
387.54094241	G	2.5803725247E-03
365.78994066	F+	2.7338094597E-03
345.25972884	F	2.8963702294E-03
325.88178927	E	3.0685973655E-03
307.59144986	D+	3.2510656601E-03
290.32766834	D	3.4443840841E-03
274.03282841	C+	3.6491978198E-03
258.65254757	C	3.8661904142E-03

A= 434

	516.11588788	C	1.9375493440E-03
	487.14852894	B	2.0527620235E-03
	459.80698294	A+	2.1748256053E-03
*	434.00000000	A	2.3041474654E-03
	409.64145171	G+	2.4411592035E-03
	386.65004369	G	2.5863180836E-03
	364.94904424	F+	2.7401085597E-03
	344.46602832	F	2.9030438934E-03
	325.13263573	E	3.0756678663E-03
	306.88434308	D+	3.2585565949E-03
	289.66024841	D	3.4523204530E-03
	273.40286788	C+	3.6576061097E-03
	258.05794401	C	3.8750986869E-03

A= 429

	510.16985230	C	1.9601315042E-03
	481.53621870	B	2.0766869887E-03
	454.50966747	A+	2.2001732230E-03
*	429.00000000	A	2.3310023310E-03
	404.92208014	G+	2.4696109425E-03
	382.19555010	G	2.6164616510E-03
	360.74456216	F+	2.7720445569E-03
	340.49752568	F	2.9368789038E-03
	321.38686804	E	3.1115148111E-03
	303.34880918	D+	3.2965351099E-03
	286.32314877	D	3.4925572881E-03
	270.25306526	C+	3.7002355516E-03
	255.08492623	C	3.9202630073E-03

A= 433

	514.92668076	C	1.9420240539E-03
	486.02606689	B	2.0575028134E-03
	458.74751985	A+	2.1798482972E-03
*	433.00000000	A	2.3094688221E-03
	408.69757739	G+	2.4467969846E-03
	385.75914497	G	2.5922911045E-03
	364.10814783	F+	2.7464367549E-03
	343.67232779	F	2.9097483827E-03
	324.38348219	E	3.0827710254E-03
	306.17723630	D+	3.2660821296E-03
	288.99282848	D	3.4602934794E-03
	272.77290736	C+	3.6660532370E-03
	257.46334045	C	3.8840481066E-03

A= 428

	508.98064519	C	1.9647112507E-03
	480.41375665	B	2.0815390612E-03
	453.45020438	A+	2.2053138147E-03
*	428.00000000	A	2.3364485981E-03
	403.97820584	G+	2.4753810615E-03
	381.30465138	G	2.6225748791E-03
	359.90386575	F+	2.7785212966E-03
	339.70382516	F	2.9437407704E-03
	320.63771450	E	3.1187847056E-03
	302.64170239	D+	3.3042372948E-03
	285.65572885	D	3.5007174687E-03
	269.62310474	C+	3.7088809617E-03
	254.49032267	C	3.9294225003E-03

A= 432

	513.73747365	C	1.9465194799E-03
	484.90360484	B	2.0622655514E-03
	457.68805675	A+	2.1848942423E-03
*	432.00000000	A	2.3148148148E-03
	407.75370309	G+	2.4524608665E-03
	384.86824625	G	2.5982917784E-03
	363.26725141	F+	2.7527942476E-03
	342.87862726	F	2.9164839115E-03
	323.63432866	E	3.0899070693E-03
	305.47012952	D+	3.2736425050E-03
	288.32540856	D	3.4683034180E-03
	272.14294684	C+	3.6745394713E-03
	256.86873690	C	3.8930389586E-03

A= 427

	507.79143807	C	1.9693124480E-03
	479.29129460	B	2.0864138599E-03
	452.39074128	A+	2.2104784840E-03
*	427.00000000	A	2.3419203747E-03
	403.03433152	G+	2.4811782069E-03
	380.41375266	G	2.6287167406E-03
	359.06276934	F+	2.7850283721E-03
	338.91012464	F	2.9506347768E-03
	319.88856096	E	3.1260886510E-03
	301.93459561	D+	3.3119755554E-03
	284.98830892	D	3.5089158702E-03
	268.99314421	C+	3.7175668657E-03
	253.89571911	C	3.9386248949E-03

A= 431

	512.54826653	C	1.9510357664E-03
	483.78114279	B	2.0670503902E-03
	456.62859366	A+	2.1899636026E-03
*	431.00000000	A	2.3201856148E-03
	406.80982877	G+	2.4581510309E-03
	383.97734754	G	2.6043202975E-03
	362.42635500	F+	2.7591812441E-03
	342.08492674	F	2.9232506955E-03
	322.88517512	E	3.0970762272E-03
	304.76302274	D+	3.2812379633E-03
	287.65798863	D	3.4763505257E-03
	271.51298631	C+	3.6830650850E-03
	256.27413334	C	3.9020715316E-03

A= 426

	506.60223096	C	1.9739352472E-03
	478.16883256	B	2.0913115451E-03
	451.33127819	A+	2.2156674007E-03
*	426.00000000	A	2.3474178404E-03
	402.09045721	G+	2.4870025689E-03
	379.52285394	G	2.6348874372E-03
	358.22187292	F+	2.7915659975E-03
	338.11642411	F	2.9575611497E-03
	319.13940742	E	3.1334268873E-03
	301.22748883	D+	3.3197501459E-03
	284.32088899	D	3.5171527620E-03
	268.36318368	C+	3.7262935485E-03
	253.30111555	C	3.9478704932E-03

A= 430

	511.35905942	C	1.9555730589E-03
	482.65868075	B	2.0718574841E-03
	455.56913056	A+	2.1950565412E-03
*	430.00000000	A	2.3255813954E-03
	405.86595446	G+	2.4638676613E-03
	383.08644882	G	2.6103768564E-03
	361.58545858	F+	2.7655979417E-03
	341.29122622	F	2.9300489528E-03
	322.13602158	E	3.1042787302E-03
	304.05591596	D+	3.2888687492E-03
	286.99056870	D	3.4844350618E-03
	270.88302579	C+	3.6916303526E-03
	255.67952978	C	3.9111461166E-03

A= 425

	505.41302384	C	1.9785798007E-03
	477.04637051	B	2.0962322780E-03
	450.27181509	A+	2.2208807358E-03
*	425.00000000	A	2.3529411764E-03
	401.14658289	G+	2.4928543397E-03
	378.63195523	G	2.6410871724E-03
	357.38097651	F+	2.7981343881E-03
	337.32272358	F	2.9645201171E-03
	318.39025389	E	3.1407996564E-03
	300.52038205	D+	3.3275613227E-03
	283.65346906	D	3.5254284155E-03
	267.73322316	C+	3.7350612979E-03
	252.70651199	C	3.9571596003E-03