The design of attenuation networks

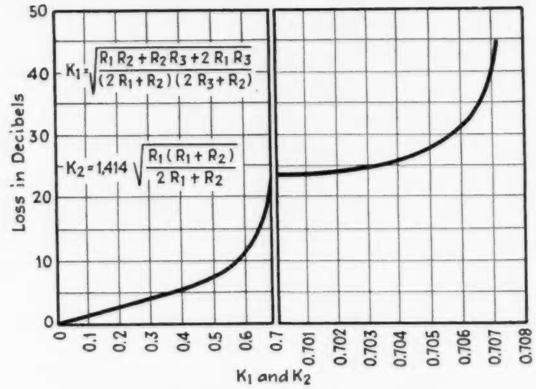

Fig. 2—Loss of either H or T type pads as a function of the resistances

By W. F. LANTERMAN

THE term "pad" as commonly employed in connection with audio frequency circuits, refers to an attenuation device used to reduce the power at a point in a circuit by some desired value. Pads are useful to the radio and sound engineer in innumerable ways. As accurately calibrated constant impedance attenuators, they are valuable in testing and measurement work. Fixed pads are used for terminating apparatus or transmission lines to provide loads of definite impedance; variable pads are often used as volume controls, and for this purpose are superior to most other devices. The following work is an attempt to put pad design formulas into simplified forms most useful in practical applications.

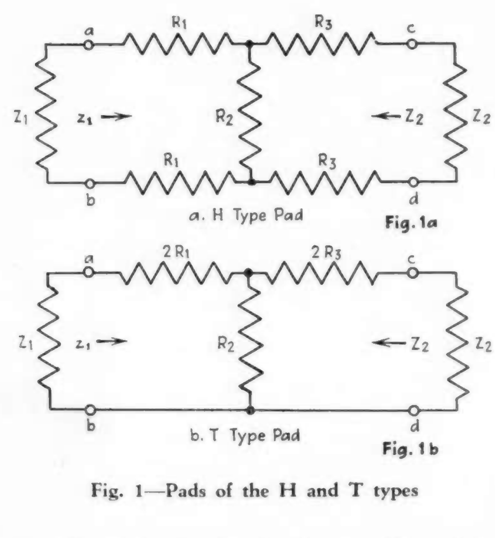

Fig. 1—Pads of the H and T types

Regarded as an electric circuit, a pad consists of a one-section artificial line whose elements are pure resistances. It is designed so that its input and output impedances are, respectively, the image impedances of the apparatus from which and into which it works: i.e., the impedances at the junction points of the pad and the other parts of the circuit are "matched." Such a condition exists if the impedance at the terminals (a, b) of the network (Fig. 1) equals the impedance Z_1 of the preceding apparatus, when the network is terminated at (c, d) by an impedance equal to Z_2. The power loss caused by the insertion of a pad is measured in decibels. When the values of the input and output impedances and the loss in *db* are designated, a pad is completely specified, and the amount of resistance required for each leg may be computed. Conversely, if all of the resistances of an existent pad are known, its input and output impedances, and its loss in *db* can be determined.

H or T types of network

The resistances forming the pad are arranged in an H or T network. The T pad is simpler and easier to construct than the H type, and in general may be used with the same results; the H type is necessary only where each side of the line is required to contain the same series impedances to preserve the electrical balance of the system. For example, a transformer circuit with center taps grounded would require pads of the H type, whereas a circuit with one side of the line grounded might use T pads. The expressions to be developed here apply to both types if the resistances have the values shown in Fig. 1.

From fundamental circuit theory we obtain

$$Z_1 = 1.414\sqrt{\frac{(2R_1 + R_2)\ (R_1R_2 + R_2R_3 + 2R_1R_3)}{(2R_3 + R_2)}} \quad (1)$$

$$Z_2 = 1.414\sqrt{\frac{(2R_3 + R_2)\ (R_1R_2 + R_2R_3 + 2R_1R_3)}{(2R_1 + R_2)}} \quad (2)$$

The loss in decibels is

$$L_{db} = 8.686 \tanh^{-1}\left[1.414\sqrt{\frac{R_1R_2 + R_2R_3 + 2R_1R_3}{(2R_1 + R_2)\ (2R_3 + R_2)}}\right] \quad (3)$$

$$= 8.686 \tanh^{-1}(1.414\ K_1) \quad (3a)$$

where $K_1 = \sqrt{\frac{R_1R_2 + R_2R_3 + 2R_1R_3}{(2R_1 + R_2)\ (2R_3 + R_2)}}$

A plot of (3a) for various values of the quantity K_1 is shown in Fig. 2.

Equations (1), (2) and (3) are the general expressions for finding the input and output impedances and the loss of a pad having known resistances. In the special case where $R_1 = R_3$, (1) and (2) reduce to the identity (4) and the impedances are equal:

$$Z_1 = Z_2 = 2\sqrt{R_1(R_1 + R_2)} \quad (4)$$

[1]The reader who is interested in the mathematical development of these expressions will find a complete treatment of the subject in K. S. Johnson's "Transmission Circuits for Telephone Communication."

Also, (3) becomes

$$L_{db} = 8.686 \tanh^{-1} \left[\frac{2\sqrt{R_1(R_1 + R_3)}}{2R_1 + R_3}\right] \qquad (5)$$

$$= 8.686 \tanh^{-1} (1.414 K_2) \qquad (5a)$$

where

$$K_2 = 1.414 \frac{\sqrt{R_1(R_1 + R_3)}}{2R_1 + R_3}$$

the values of L_{db} in (5a) corresponding to various values of K_2 can be read from the curve, Fig. 2.

To design a pad

In practice, the conditions are usually the reverse of those just considered—the engineer is most often concerned with designing a pad to have given input and output impedances and to produce a given loss. The equations for this purpose are developed from (1), (2) and (3) by a somewhat involved mathematical process.[1] The results are the general equations for a transmission network:

$$R_1 = \frac{1}{2}\left[\frac{Z_1}{\tanh \theta} - \frac{\sqrt{Z_1 Z_2}}{\sinh \theta}\right] \qquad (6)$$

$$R_3 = \frac{1}{2}\left[\frac{Z_2}{\tanh \theta} - \frac{\sqrt{Z_1 Z_2}}{\sinh \theta}\right] \qquad (7)$$

$$R_2 = \frac{\sqrt{Z_1 Z_2}}{\sinh \theta} \qquad (8)$$

in which

$$\theta = \frac{1}{2} \log_e \frac{\text{volt-amperes at } (a, b)}{\text{volt-amperes at } (c, d)} \qquad (9)$$

For resistance networks, (9) may be written

$$\theta = \frac{\text{loss in } db}{8.686}$$

For practical purposes, we may rewrite equations (6), (7) and (8) as follows:

$$R_1 = K_3 Z_1 - K_4\sqrt{Z_1 Z_2} \qquad (10)$$

$$R_3 = K_3 Z_2 - K_4\sqrt{Z_1 Z_2} \qquad (11)$$

$$R_2 = 2K_4\sqrt{Z_1 Z_2} \qquad (12)$$

where K_3 and K_4 depend on the loss in *db* as shown in Table I. These are the relations most useful in the practical design of pads.

From (10) and (11) we see that if $K_4\sqrt{Z_1 Z_2}$ is larger than $K_3 Z_1$ or $K_3 Z_2$, R_1 or R_3 will be negative. Since this is not possible practically, there is a limit to the ratio of $\frac{Z_1}{Z_2}$ or $\frac{Z_2}{Z_1}$ consistent with any given loss in *db*. This limit is reached when R_1 or R_3 becomes zero; it is found from (10) and (11) to be

$$\frac{Z_1}{Z_2} \text{ or } \frac{Z_2}{Z_1} \leq \frac{K_3^2}{K_4^2} = K_5 \; (= \cosh^2 \theta) \qquad (13)$$

where Z_1 and Z_2 are taken in the order which makes the ratio $\frac{Z_1}{Z_2}$ or $\frac{Z_2}{Z_1}$ equal to or larger than unity. In other words, the ratio of the larger terminal impedance of the pad to the smaller impedance cannot be greater than a quantity (K_5 in equation (13)) which depends upon the loss. Values of K_5 for various values of the loss in *db* are given in Table I. The limiting values of the ratios are plotted in Fig. 3.

From equations (10), (11) and (12) together with (13) and Table I, we can determine the resistance values for any pad of given loss and input impedances. To illustrate the method of applying them to a practical design problem, assume that we wish to build a pad to work between a circuit whose terminal impedance is 400 ohms and another whose input impedance is 600 ohms,

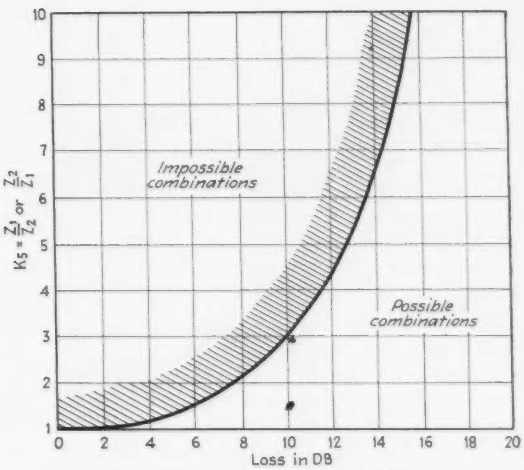

Fig. 3—Loss as a function of the terminal impedances

and that the pad so designed is to have a loss of 10 *db*.

First, referring to Fig. 3 or Table I, we find it is possible to construct a pad having this loss and an impedance ratio of $\frac{600}{400}$ or 1.5 (3.03 being the maximum impedance ratio possible). Then by Table I we see that for a loss of 10 *db*,

TABLE I

Loss in db	K_3	K_4	K_5
1	4.34	4.34	1.013
2	2.21	2.15	1.05
3	1.51	1.43	1.12
4	1.16	1.05	1.23
5	.965	.820	1.37
6	.835	.670	1.56
7	.725	.525	1.79
8	.690	.476	2.10
9	.645	.406	2.50
10	.610	.352	3.03
12	.565	.269	4.45
14	.540	.208	6.76
15	.532	.184	8.35
16	.525	.163	10.43
18	.515	.128	16.74
20	.510	.101	25.40
25	.502	.056	79.80
30	.500	.0318	247.00
35	.500	.0178	784.00
40	.500	.0100	2401.00
45	.500	.00565	7921.00
50	.500	.00320	24964.00

$$K_3 = 0.610$$
$$K_4 = 0.352.$$

Substituting these and the given values of Z_1 and Z_2 in (10), (11) and (12) gives

$$R_1 = (.61 \times 400) - (.352 \times 490) = 71.5 \text{ ohms.}$$
$$R_2 = 2 \times .352 \times 490 = 345 \text{ ohms.}$$
$$R_3 = (.61 \times 600) - (.352 \times 490) = 193.5 \text{ ohms.}$$

In practice we would make $R_1 = 70$ ohms, $R_2 = 350$ ohms, and $R_3 = 200$ ohms.

Thus far only fixed pads have been dealt with. In many applications, of which the "gain" or volume control is the best example, a pad having a variable loss is needed. A pad might also be designed to have variable input and output impedances, but there is little practical application for such an arrangement in ordinary audio frequency circuits.

To vary the amount of loss in a pad and at the same

[*Continued on page 532*]

The design of attenuation networks

[*continued from page 509*]

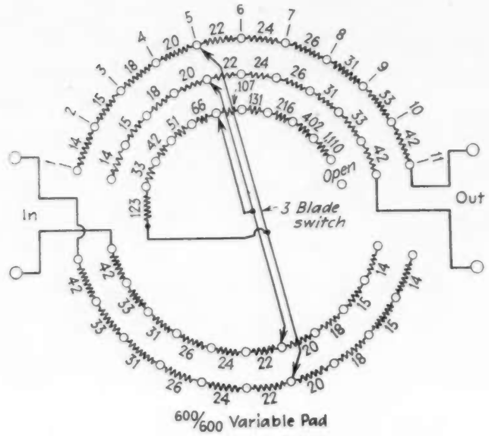

Fig. 6—Circuit and switching arrangement for variable pad

time maintain constant input and output impedances, means that all of the component resistances must be made variable. They may be either continuously so, as for instance, the graphite- element sliding contact type, or may be fixed resistances with taps. The latter method is preferable from the standpoint of permanence of calibration and mechanical construction. It has the disadvantage that the pad is adjustable only by steps instead of continuously; but if these steps are made small enough (say one or two *db* each) the change in volume per step will be scarcely noticeable at the loud speaker.

The labor in the design of a variable pad is greatly reduced by plotting curves for the expressions (10), (11) and (12). In the majority of practical cases, Z_1 equals Z_2, so that (10) and (11) become identities and but two curves are required. Such curves are shown in Figs. 4 and 5. From them the ratios of $\frac{R_1}{Z_1}, \frac{R_3}{Z_1}$ and $\frac{R_2}{Z_1}$ for any loss in *db* can be read directly, and the values of R_1, R_3 and R_2 subsequently obtained by multiplying these ratios by Z_1. Designing a variable pad actually consists of designing a series of fixed pads. Taps and switch contacts are arranged so that on any step each resistance is increased or decreased from its value on the preceding step by the required amount.

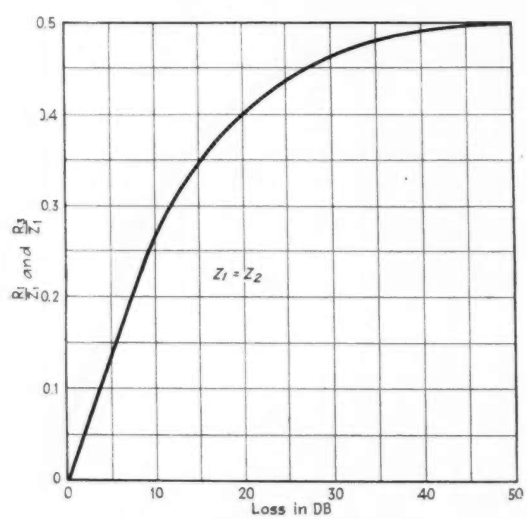

Fig. 4—Loss as a function of ratio between shunt resistance

Let us design, as an example, a pad with input and

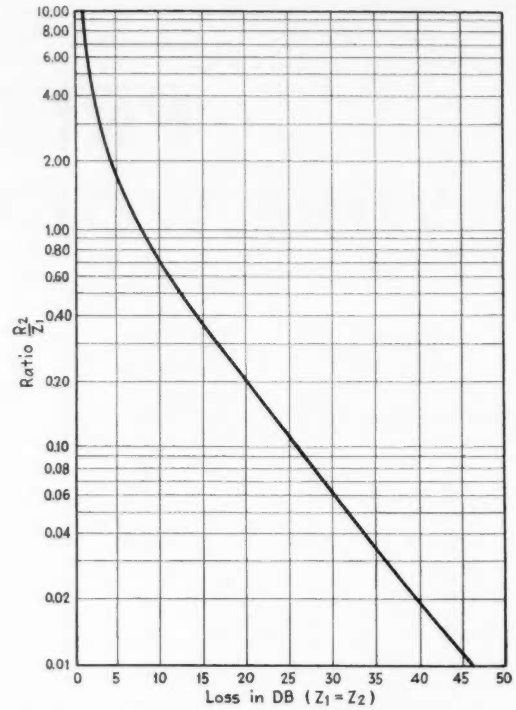

Fig 5—Loss as a function of the shunt resistance

output impedances of 600 ohms and a loss range of from 20 to 0 *db* in eleven steps of 2 *db* each. The first step will correspond to minimum volume or the maximum loss of 20 *db*. From Figs. 4 and 5, we find for 20 *db*, $\frac{R_1}{Z_1} = 0.408$ and $\frac{R_2}{Z_1} = 0.205$. Since Z_1 and Z_2 are 600 ohms, R_1 and R_3 are each 245 ohms, and R_2 is 123 ohms. Likewise, for the second step, which has 18 *db* loss, $R_1 = 231$ and $R_2 = 156$, and so on for each step, giving the results for the complete pad shown in Table II. The last step has a zero loss and R_1 and R_3 will be zero, while R_2 will be infinite or open circuited.

TABLE II

Design of variable pad having 11 steps of 2 db each, from 0 to 20, and output and input impedances of 600 ohms.

Step	Loss db	R_1/Z_1	R_1 and R_3	ΔR_1 and ΔR_3	R_2/Z_1	R_2	ΔR_2
1	20	.408	245	....	.205	123	....
2	18	.386	231	—14	.260	156	33
3	16	.360	216	—15	.330	198	42
4	14	.330	198	—18	.415	249	51
5	12	.296	178	—20	.525	315	66
6	10	.258	155	—22	.702	421	107
7	8	.220	132	—24	.920	552	131
8	6	.176	106	—26	1.280	768	216
9	4	.125	75	—31	1.950	1170	402
10	2	.070	42	—33	3.80	2280	1110
11	0	.000	0	—42	Open circuit		