An Introduction to Gyrator Theory

How inductors can be simulated using resistors, capacitors, and op amps.

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A GYRATOR, believe it or not, is an inductor without any turns of wire. Although the theory behind this interesting circuit has been established for some time, only within the past few years have synthesized inductors been used on a wide scale. Before we examine the gyrator in detail, let's review some basic properties of inductors.

A pure inductance is a circuit element whose opposition to the flow of alternating current (*inductive reactance*) varies directly with frequency. At dc or zero hertz, the ideal inductor has zero ohms of resistance (a perfect conductor) and zero ohms of reactance. Therefore, we can say that it also has zero ohms of impedance—the vector sum of resistance and reactance. However, as we move into the realm of ac, the reactance of an inductor increases according to the formula $X_L = 2\pi fL$; where X_L is measured in ohms; f (frequency) in hertz; and L (inductance) in henries. Its resistance remains zero ohms. At infinite frequency, the inductor has infinite reactance, and will permit no ac to flow.

So far we have been talking about an *ideal* inductor. Actually, every inductor has a certain amount of resistance and capacitance as well as inductance. As shown in Figs. 1A and 1B, an iron-core inductor can be modeled as an inductance in series with a resistance, *R1*; and this combination is in parallel with a capacitance and series resistance, *R2*. An air-core inductor (Figs. 2A and 2B) behaves as an inductance and series resistance *R1* would. In both cases, *L* is the inductance of the coil, and *R1* is the resistance of the wire which comprises the coil. The iron-core inductor contains two additional elements, *R2* and *C*, which represent losses within the core. With dc, there are no core losses, and consequently, our model's *C* permits no current to flow through *R2*. At higher and higher frequencies, core losses increase. Thus, in our model, increased current flows through *R2* as the capacitor's reactance decreases.

Synthesizing an Inductor. By combining resistors and a capacitor with a gain stage, we can create a circuit which appears to the "outside world" as a real inductor. To understand how, we will analyze the inductor models (Figs. 1B and 2B) in terms of "port admittance." A *port* is a point through which energy can enter or leave. In the case of an electrical circuit, it can consist of a pair of terminals to which a circuit element is connected. The inductors and their models in Figs. 1 and 2 are ports, and when a voltage source is connected across them, an imput voltage (V_{IN}) is applied an an input current (I_{IN}) flows.

Admittance, measured in *mhos*, is the reciprocal of impedance. In other words, admittance is the ratio of current to voltage. If an element's admittance is zero mhos, no current will flow through it no matter how high the voltage is across it. Such an element is a perfect insulator or open circuit. On the other hand, an element with infinite admittance will conduct infinite current, even if a low voltage source is connected across it. It is a perfect conductor or a short circuit. Combining these two terms, port admittance is the ratio of the current flowing into the port (I_{IN}) to the voltage across the port (V_{IN}).

Referring to Fig. 1B, we can see that resistors *R1* and *R2* set the limits of port impedance at both very high and very low frequencies. At dc, the admittance of the inductor *L* is infinite (a short circuit), and only *R1* limits the current through it. Capacitor *C* behaves as an open circuit with zero admittance, so *R2* is removed from the circuit. At an infinite frequency *L* is an open circuit and *R1* is removed from the circuit. However, *C* is a short circuit and current through it is limited only by *R2*. Between these frequency extremes, *L* will determine the port's admittance, because it is much larger than *C*.

The port admittance of the air-core coil at dc is simply the reciprocal of resistance *R1*, since *L* has infinite admittance. At an infinite frequency, the port admittance is zero, because the inductance acts as an open circuit, and no input current can flow.

Analyzing the Gyrator. Now let's apply these concepts to the gyrator circuits (Figs. 1C and 2C). As in the equivalent circuits, *R1* represents the ohmic resistance of the coil wire, and *C* and *R2* are core losses which increase in step with the applied frequency. However, something new has been added—a gain stage. Any active device can be used, but here we choose an op amp for its simplicity, high gain, almost infinite input impedance, and very low output impedance. The gyrator op amps are strapped for unity-gain, noninverting operation. So, within the frequency limits of the device (assume infinite bandwidth), the voltage at the output is exactly the same as that at the noninverting input.

If we apply a dc voltage across the input terminals of Fig. 1C, capacitor *C*

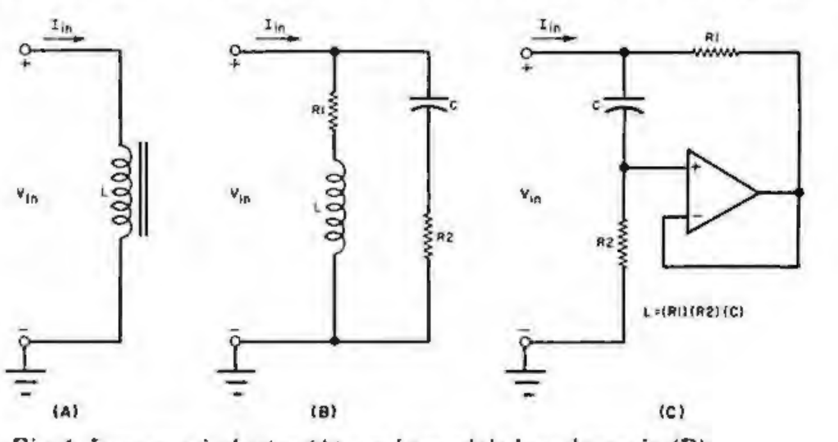

Fig. 1. Iron-core inductor (A) can be modeled as shown in (B) and simulated using the gyrator circuit in (C).

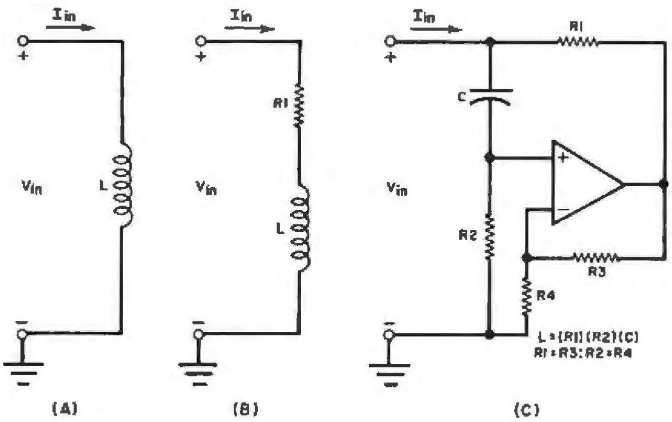

Fig. 2. An air-core coil (A) has an equivalent circuit shown in (B). Op amp gyrator (C) simulates the coil's behavior.

does not conduct, and the voltage at the noninverting input is zero. The output is also at ground potential, and because the op amp has very high output admittance (low output impedance), we can safely say that *R1* is connected across the port. So, I_{IN} will flow only through *R1*. This agrees with the behaviour of the equivalent circuit of Fig. 1B. The port admittances are maximized at dc, limited only by the values of both *R1*'s (assumed to be equal).

At infinite frequency, *C* is a short circuit, and therefore the voltage at the op amp's noninverting input (as well as that at the output) is equal to V_{IN}. Since there is no voltage drop across *R1*, it is effectively removed from the circuit. The only admittance path is through *R2* to ground, which is the same behavior we noted in the equivalent circuit.

For frequencies between zero and infinity, *C* and *R2* act as a high-pass filter, causing less and less voltage drop across *R1* as frequency increases, and thus less port admittance until *R2*'s limiting effect comes into play. The reactive characteristics of the capacitor have successfully been inverted or gyrated so that the port behaves as an inductor. The equivalent inductance in henries is expressed by the formula $L = (R1)(R2)(C)$, with resistances in ohms and capacitance in farads.

With the addition of two resistors, an air-core inductor can be simulated. Air-core coils have essentially no "core" loss, and therefore have no parallel resistance in their equivalent circuits. Because of this the gyrator (Fig. 2C) uses the additional resistors to set the gain of the op amp. When the values are properly selected, they provide enough gain to compensate for *R2*'s losses at high frequencies. But the amount of gain must be carefully chosen—otherwise the circuit might oscillate! If *R3* equals *R1* and *R4* equals *R2*, the circuit will be stable and exhibit no parallel resistance. In practice, however, little is gained over the circuit of Fig. 1C as long as the ratio *R2/R1* is at least 90 to 100, because the effects of parallel resistance are negligible in most audio applications commonly encountered.

Practical Design. In synthesizing a useful "inductor," the same basic rules that govern the optimization of wound coils should be followed. For example, series resistance *R1* should be kept as small as possible and parallel resistance *R2* as large as possible. This corresponds to a coil wound from the heaviest wire practicable on the least lossy core available. For best performance, *R1* should be no lower than the op amp's minimum recommended load resistance, which falls between 100 and 2000 ohms for common op amp types. The largest acceptable value for *R1* is desirable, so as not to load the op amp too much, thus preventing high distortion and heating effects. To simulate a high-quality toroidally wound coil, *R2* should be at least 100 times greater than *R1*, but not so large as to become a major contributor to the op amp's input noise. As a rule of thumb, keep *R1* around 1000 ohms and *R2* between 10 kilohms and 1 megohm.

Once the values of *R1* and *R2* have been chosen, use the formula $C = L/(R1)(R2)$ to find the required capacitance in farads. At least 100 pF should be used to avoid the detuning influences of stray capacitances.

It is important to keep the op amp functioning within acceptable circuit and signal parameters. If for any reason it begins to deviate from the role of a voltage follower, the "inductor" won't work properly. Input signals must lie within the operating bandwidth of the device, and their amplitudes must not cause the output stages to clip. In a gyrator, clipping in the gain stage is analogous to core saturation, which can cause high distortion levels.

However, this is not usually a problem with gyrators. Because they will most often be operated from the same power supplies that other audio stages use, they will not start to clip until the other amplifiers do. Unlike iron-core coils, whose saturation characteristics are functions of core material, size, number of turns, and applied current, the gyrator's saturation point is accurately predictable, and does not occur before the other active stages of the system also saturate or clip.

Using either of the gyrators we have examined will result in high-quality coils with inductances ranging from millihenries to hundreds or thousands of henries. Commonly available parts—including relatively small capacitors—can be employed. Added benefits include high magnetic field immunity and saturation characteristics, and (paradoxically) small amounts of required printed circuit board "real estate." However, there is one limitation. The gyrators we have described are single ended. That is, one side is grounded. To simulate "floating" inductors, neither side of which is connected to ground, more complex circuits using two op amps can be designed. But such gyrators are beyond the scope of this article. ◇

PROPERTIES OF GYRATORS

Advantages

1. Immunity to ambient magnetic fields; no coupling or crosstalk between "inductors."
2. Very small size required for large values of inductance.
3. Inexpensive, use readily available components.
4. Accurately predictable "saturation" levels.
5. Parameters can be fixed by choice of resistors.

Disadvantages

1. Active device generates noise (can be held to low levels if proper devices are selected).
2. More complex circuits are required to simulate "floating" inductors.
3. Inductors with low series resistance and high current handling characteristics are difficult and impractical to simulate, as the circuits require high-power active devices.
4. Simulated inductors are frequency limited by their active devices' usable bandwidths and slew rates (not a problem at audio frequencies in most cases).