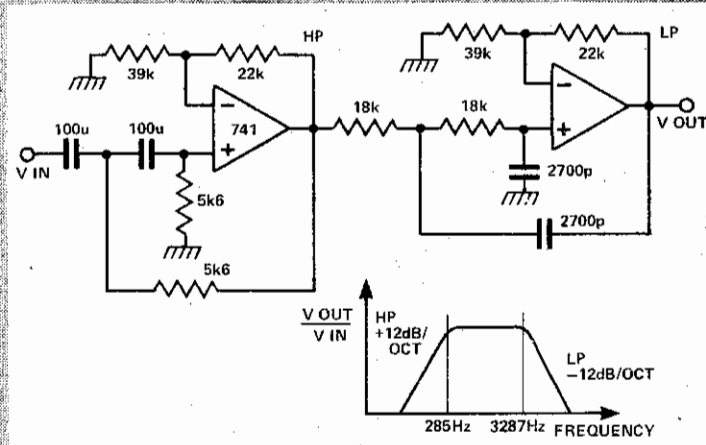


## Simple Speech Filter

The telephone system has been designed for speech communication. The bandwidth of the system is 300 Hz to 3400 Hz, which has been arrived at after many years of experimentation. Thus, it is true to say that much of the information in speech is contained between these frequency limits. The circuit shows a filter structure that will simulate the telephone bandwidth. It could have many uses, for instance as a 'speech filter' for noisy radio reception or land line communications, or as a voice detector for a light show.



# How to build high-quality filters out of low-quality parts

If the operational amplifier is wired as a voltage follower, an active filter can use loose-tolerance resistors and capacitors—and still perform well

by Philip R. Geffe, *Lynch Communication Systems Inc., Reno, Nev.*

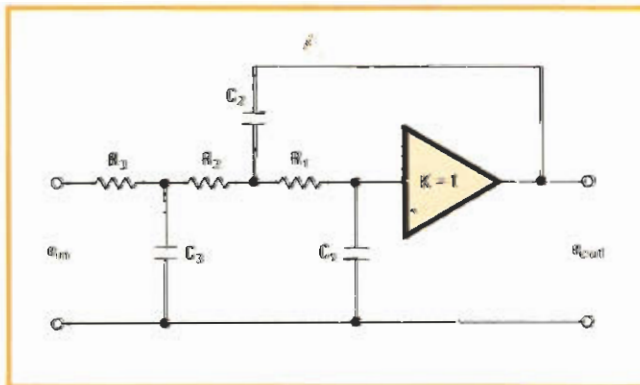
□ An active filter's sensitivity to component values is an important factor in determining its cost. If the circuit is designed for minimum sensitivity to these values, then inexpensive low-tolerance components can be used without harm to its performance.

In the three-pole filter of Fig. 1, the amplifier closed-loop gain,  $K$ , is the parameter that governs sensitivity. This results from the presence of the factor  $(1 - K)$  in terms of the transfer-function denominator. If  $K$  is greater than unity, the denominator will contain negative terms that lead to high sensitivity. For this reason, the amplification is provided by an operational amplifier connected as a voltage follower. In addition to providing  $K = 1$ , use of a voltage follower also saves two gain-fixing resistors in the op-amp connection.

## Low-pass design

If the amplifier in Fig. 1 is a voltage follower and the resistor values are all 1 ohm, a normalized Butterworth or Chebyshev filter response is obtained by choosing capacitance values from Table 1. The zero-ripple design is the normalized Butterworth filter. It has an attenuation of 3 decibels at a frequency of 1 radian per second, and its asymptotic slope in the stopband is 18 dB per octave. The exact attenuation at any frequency  $\omega$  is given by the formula:

$$e_{out}/e_{in} = 1/(1 + \omega^6)^{1/2}$$



**1. Uses loose-tolerance components.** The performance of this three-pole low-pass active filter is insensitive to values of resistors and capacitors if the amplifier gain,  $K$ , is unity. Therefore inexpensive components with relatively loose tolerances can be used.

TABLE 1: ELEMENT VALUES FOR NORMALIZED THREE-POLE ACTIVE FILTERS

Ripple (dB)	$C_1$	$C_2$	$C_3$
0	0.20245	3.5468	1.3926
0.01	0.091294	2.5031	0.84044
0.03	0.097357	3.3128	1.0325
0.10	0.096911	4.7921	1.3145
0.30	0.085819	7.4077	1.6827
1.00	0.05872	14.784	2.3444

TABLE 2: ATTENUATION CONSTANTS FOR CHEBYSHEV FILTERS

Ripple (dB)	Ripple factor ( $\epsilon^2$ )
0.01	0.00230524
0.03	0.00693167
0.1	0.023293
0.3	0.0715193
1.0	0.258025

TABLE 3: BAND EDGE FREQUENCY  $\omega_c$  AS A FUNCTION OF RIPPLE AND BAND EDGE ATTENUATION  $\alpha$

Ripple (dB)	Band edge frequency $\omega_c$ (rad/s)		
	$\alpha = 1$ dB	$\alpha = 2$ dB	$\alpha = 3$ dB
0	0.798355	0.914491	1.00000
0.01	1.56352	1.74229	1.87718
0.03	1.36673	1.50770	1.61524
0.10	1.20154	1.30707	1.36899
0.30	1.08934	1.16726	1.22906
1.00	1.00000	1.05219	1.09487

The capacitance values corresponding to the nonzero values of ripple in Table 1 yield Chebyshev filters. These filters have equiripple passbands, with the edge of the ripple band at 1 rad/s. The attenuation from the Chebyshev filter designs can be calculated from the voltage-ratio formula:

$$e_{out}/e_{in} = 1/[1 + \epsilon^2(4\omega^3 - 3\omega)^2]^{1/2}$$

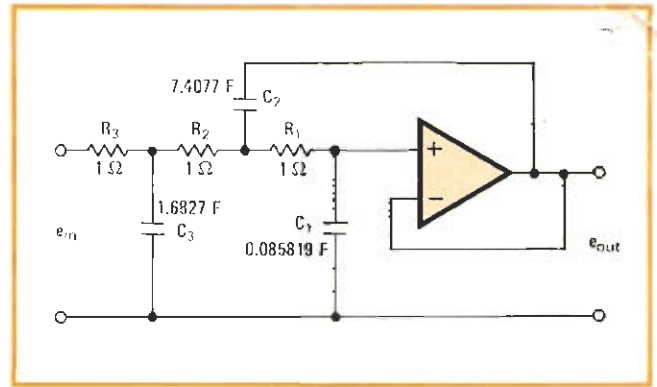
where  $\epsilon^2$  is the ripple factor obtained from Table 2. The user can be expected to define the attenuation at the edge of the passband, but the choice of ripple value is usually left to the designer. As an aid to meeting the passband edge requirement, Table 3 gives the frequencies for attenuations of 1, 2, and 3 dB for all ripple values.

As an example, consider the design of a low-pass Chebyshev filter with 0.3-dB ripple and the calculation of its attenuation at twice the 3-dB frequency.

### Frequency scaling

The normalized circuit is shown in Fig. 2, with capacitance values taken from Table 1. Table 2 shows that a 0.3-dB ripple corresponds to  $\epsilon^2$  of 0.0715193. The 3-dB frequency of this filter is given by Table 3 as 1.22906

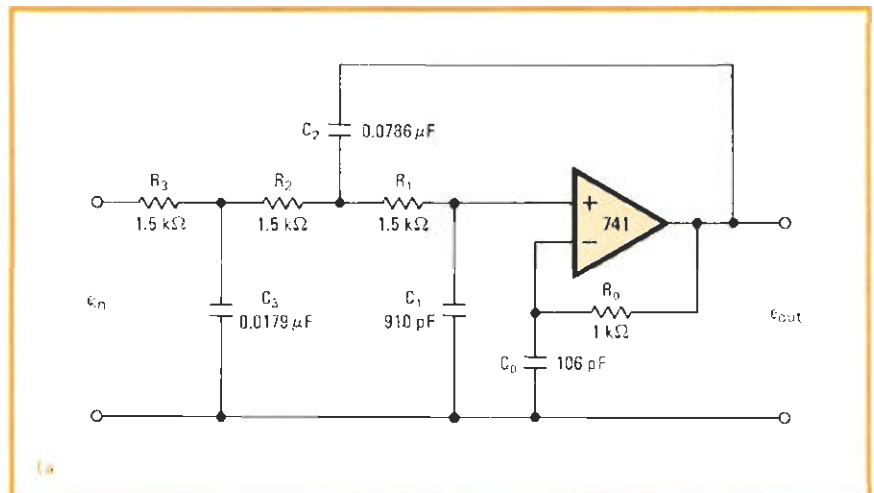
**3. Practical 10-kHz low-pass filter.** The example in the text shows how the component values in Fig. 2 are scaled to provide the edge of the ripple band (the highest frequency at which attenuation is equal to or less than 0.3 dB) at 10 kHz. Bode compensation is added to voltage follower that uses a type 741 operational amplifier.



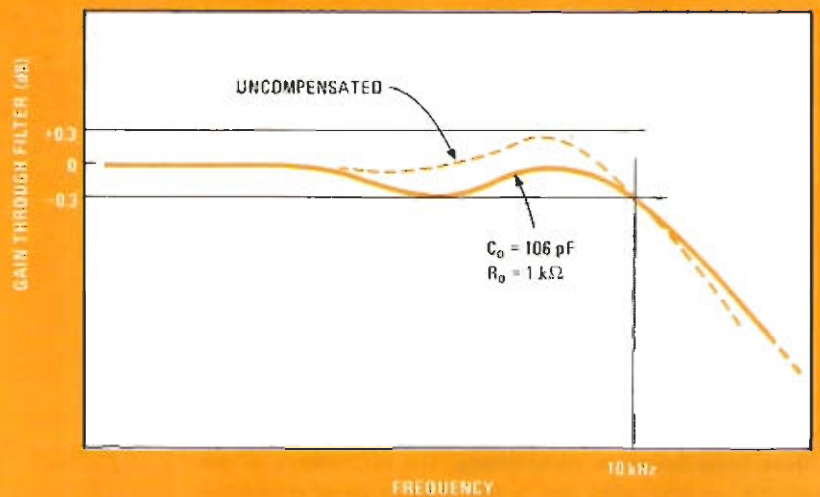
**2. Normalized low-pass filter.** Design of low-pass Chebyshev filter with 0.3-dB ripple starts with R values of 1 ohm and C values from Table 1, plus a voltage-follower op amp to make  $K = 1$ .

rad/s; at twice this frequency, i.e.,  $\omega = 2.45812$  rad/s, the attenuation is calculated to be 0.0716734. Expressed in decibels, this attenuation is  $20 \log(0.0716734)$ , or  $-22.89$  dB.

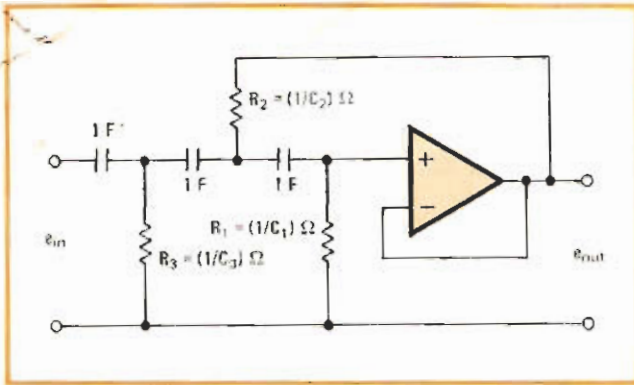
To scale the design so that the edge of the ripple band (that is, the highest 0.3-dB frequency) occurs at 10 kilohertz, divide all capacitance values by  $2\pi(10,000)$ .



(a)



(b)



**4. High-pass filter.** In the normalized high-pass three-pole active filter, the series capacitors are all 1 F, and the resistor values (in ohms) are found by reciprocating the numbers in Table 1.

This gives:

$$C_1 = 1.36585 \mu\text{F}$$

$$C_2 = 117.897 \mu\text{F}$$

$$C_3 = 26.7888 \mu\text{F}$$

These values are correct but impractical. A practical design can be obtained by multiplying all impedances by a suitable scale factor, say 1,500. This gives:

$$R_1 = R_2 = R_3 = 1,500 \Omega$$

$$C_1 = 910.6 \text{ pF}$$

$$C_2 = 0.0786 \mu\text{F}$$

$$C_3 = 0.0179 \mu\text{F}$$

The frequency response of this filter is the same as that of the normalized design, except that all attenuations are referred to the band-edge frequency,  $f_c$ , of 10 kHz, instead of to 1 rad/s. Consequently, the attenuation formulas serve for the denormalized filters if  $\omega$  is replaced by  $f/f_c$ .

#### Gain compensation

These filters require rather precise values of closed-loop gain, which is why an operational amplifier is used instead of some simpler unity-gain amplifier. But even an op amp may require compensation if its open-loop gain drops to less than 60 dB at the band edge of the filter.

A voltage divider consisting of resistor  $R_o$  and capacitor  $C_o$  provides the Boctor compensation, as it is called, by shunting off part of the feedback at high frequencies.  $R_o$  and  $C_o$  are given by:

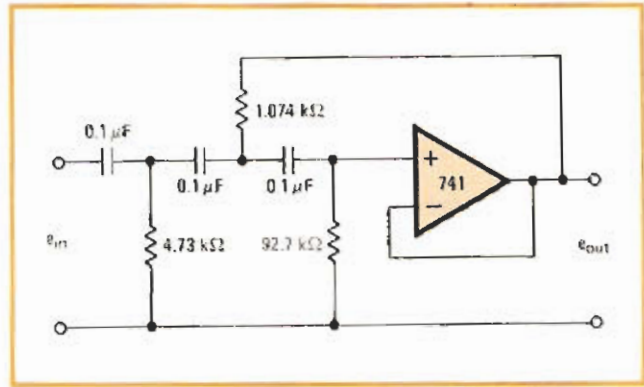
$$R_o C_o = 1/(2.6 \times 2\pi f_o)$$

where  $f_o$  is the 0-dB frequency of the amplifier open-loop gain curve.

For a 741 op amp, the open-loop gain is less than 40 dB at 10 kHz, and the measured value of  $f_o$  is 581 kHz. Therefore,  $R_o$  and  $C_o$  are 1 kilohm and 106 picofarads, respectively. The complete circuit for the 10-kHz filter in the example, including Boctor compensation, is shown in Fig. 3.

#### High-pass design

The element values given in Table 1 are the reciprocals of the resistances in a normalized high-pass filter, as



**5. Practical 200-Hz high-pass filter.** The example in the text gives details of the design of this filter, which has 0.3-decibel ripple above 200 hertz and 31.35-dB attenuation at 60 Hz.

### How to design three-pole filters

The following equations will yield component values for three-pole active filters not covered by Table 1. Given:

$$e_{out}/e_{in} = 1/(a_3 s^3 + a_2 s^2 + a_1 s + 1)$$

with known numerical coefficients  $a_3, a_2, a_1$ , component values for the circuit in Fig. 1 are obtained by finding a positive real root of:

$$18x^3 - 12a_1 x^2 + (2a_1^2 + 3a_2)x + (2a_3 - a_1 a_2) = 0$$

If  $x_0$  is such a root, then:

$$y = -a_3/(3x_0^2 - a_1 x_0)$$

$$z = a_1 - 3x_0$$

Then the element values for Fig. 1 are  $K = 1, R_1 = R_2 = R_3 = 1, C_1 = x_0, C_2 = y,$  and  $C_3 = z,$  where resistances are in ohms and capacitances are in farads.

shown in Fig. 4. The normalized capacitance values for the high-pass filter are all 1 farad. In the attenuation formula used for frequency-response calculations,  $\omega$  must be replaced by  $f_c/f$ , where  $f_c$  is the band-edge frequency; this applies to both normalized and denormalized highpass filters.

As an example, consider the design of a high-pass filter that must pass signals above 200 hertz and must suppress 60-Hz signals by at least 30 dB. Here  $f_c/f$  is 200/60; the voltage-ratio expression shows that a Chebyshev filter with 0.3-dB ripple will give an attenuation of 31.35 dB. Therefore, the resistors in the normalized circuit (Fig. 4) have values found from Table 1 as follows:

$$R_1 = 1/C_1 = 1/0.085819 = 11.6524 \Omega$$

$$R_2 = 1/C_2 = 1/7.4077 = 0.1350 \Omega$$

$$R_3 = 1/C_3 = 1/1.6827 = 0.59428 \Omega$$

Frequency-scaling this design to  $\omega_c = 2\pi(200 \text{ Hz})$  gives 1256.6 for  $\omega_c$ , and dividing the 1-F capacitances by this number gives the value  $C_1 = C_2 = C_3 = 795.8 \mu\text{F}$  for the capacitors.

Finally, to get practical component values, all the impedances are multiplied by the factor 7,958 to make the capacitors 0.1  $\mu\text{F}$ . The resistors then turn out to be as shown in Fig. 5.  $\square$

# Active filter has stable notch, and response can be regulated

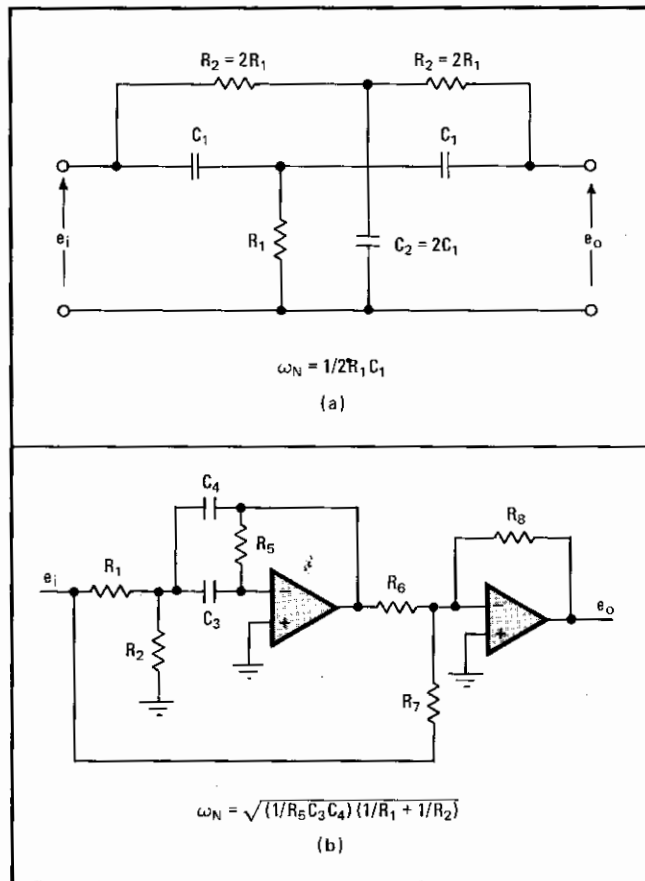
High Q of zeroes in transfer function is independent of component balance; notch depth depends on high gain, rather than precision of parts, and circuit-performance sensitivity to passive elements is low

by James R. Bainter, *Motorola Semiconductor Products Inc., Phoenix, Ariz.*

Many tone-signalling systems require elimination or rejection of a single frequency or a narrow range of frequencies. To produce this stop band, transfer functions with a notch response have been achieved by both active and passive networks. However, all the circuits that have been used in the past have required accurately matched component values to produce deep notches. Unfortunately, aging and temperature variations can affect the capacitances or resistances of carefully matched components differently, with the result that

their match is degraded. Aging can thus reduce a rejection ratio of, say, 60 decibels to as little as 10 dB.

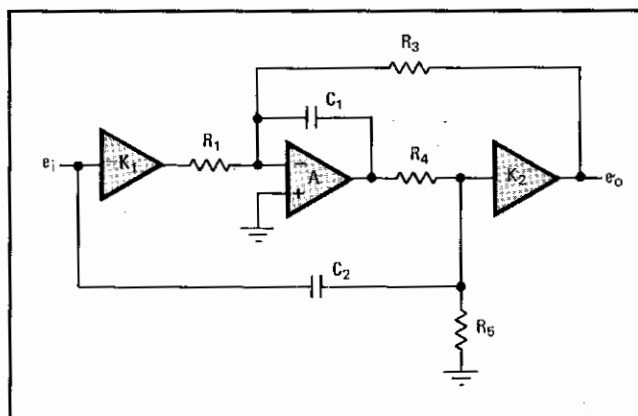
Now this need for perfect matching has been eliminated by an active filter in which the Q, or sharpness, of the null is a function of amplifier gain, rather than of precision balancing of passive components. The notch depth in the new network is constant so long as the gain remains high, even if resistors and capacitors drift. The active-filter network can also generate low-pass or high-pass filter blocks for frequencies above or below the rejection frequency. The passive component sensitivities of the network are 0.5 or less.



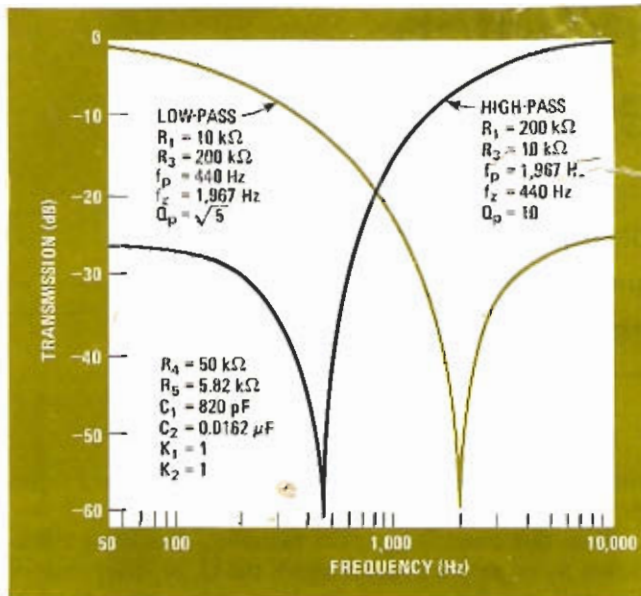
**1. Null networks.** Conventional notch filters may be passive circuits such as the bridged-T network in (a) or active circuits such as the subtractive arrangement in (b). In either type, the amount of signal rejection at the notch frequency depends upon ratios of passive components; therefore, component drift degrades performance.

## Considering null networks

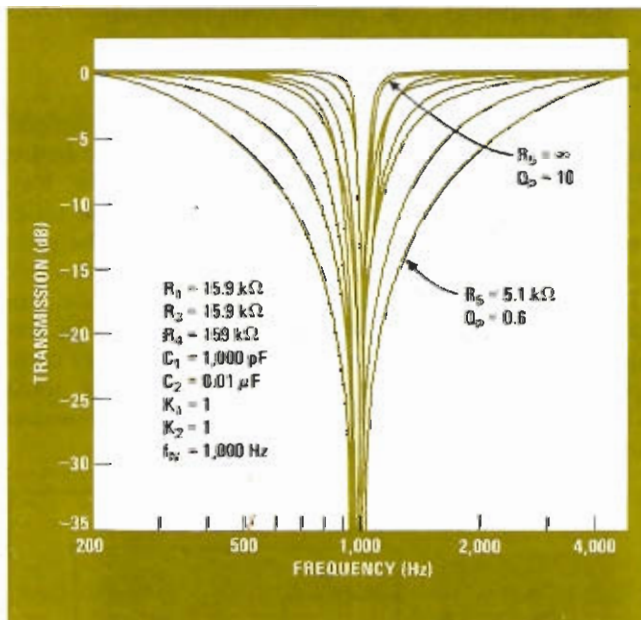
Among the passive bandstop networks that depend on the precision with which components are matched is the symmetrical parallel-T network shown in Fig. 1(a). One condition for balance is that the ratio of the series and shunt capacitors must be proportional to the ratio of the series and shunt resistors ( $C_2/C_1 = 4R_1/R_2$ ). This balance, which is independent of frequency, implies that the depth of the notch at the rejection frequency depends solely on the accuracy of passive component matching. To get 60 dB of rejection at the notch frequency, the ratio of  $C_1$  to  $C_2$  must be held within 0.1% over the temperature range of interest.



**2. Zero generation.** New circuit can generate true zeroes at any frequency for which op amp has high gain. Notch depth is function of this gain alone, not of component ratios. Filter can combine low-pass or high-pass characteristics with notch. Pole and zero frequencies and pole Q-factor are independently adjustable.



**3. Characteristics.** Interchanging  $R_1$  and  $R_3$  changes filter response from low-pass to high-pass. If  $R_1$  is equal to  $R_3$ , filter has symmetrical notch characteristic. Quality factor  $Q_p$ , which is measure of how fast the response returns from the notch to its passband characteristic, depends on values of resistances and capacitances.



**4. Adjustable.** Response of unity-gain notch filter is varied by varying  $R_5$  to change  $Q_p$  value. Notch frequency is 1 kHz.

Other null networks have active circuits that subtract one signal from another to produce a notch at the desired rejection frequency. As an example, Fig. 1(b) shows a multiple-feedback bandpass active filter connected to a summing amplifier. At the node of  $R_6$  and  $R_7$ , the input signal is subtracted from the output of the filter section. The final transfer function is

$$\frac{e_o(s)}{e_i(s)} = -\frac{R_8}{R_7} \left[ \frac{s^2 + s(1/C_2 R_5 + 1/C_4 R_5 - R_7/C_4 R_1 R_6) + \omega_N^2}{s^2 + s\omega_N/Q_p + \omega_N^2} \right]$$

where  $\omega_N$  is  $2\pi$  times the notch frequency, and  $Q_p$  determines notch width. To produce a deep notch with this

circuit, the middle term in the numerator of the equation must be zero. That is,

$$R_5/R_1 = (R_6/R_7)(1 + C_4/C_3)$$

This expression shows that the amount of rejection at the notch frequency depends upon three ratios of passive components. Therefore, to maintain good notch depth, these ratios must be accurately set and maintained over the range of operating temperature.

Other bandpass filters, such as the state variable or biquad, may also be used; but they also require balancing of components, because the filter sections do not inherently generate transfer zeroes.

### Generating transfer zeroes

The general form for the transfer function of an active filter is

$$\frac{e_o(s)}{e_i(s)} = \frac{s^2 + (\omega_z/Q_z)s + \omega_z^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

where  $\omega_z$  and  $\omega_p$  are the radian frequencies for the zeroes and poles, and  $Q_z$  and  $Q_p$  are the corresponding quality factors. For infinite  $Q_z$ , the coefficient of  $s$  in the numerator would be zero; this fact suggests that a circuit with a large gain factor in the denominator of this  $s$ -coefficient must have high  $Q_z$ . Such a circuit is shown in Fig. 2. The coefficient of  $s$  in the numerator of its transfer function is

$$\omega_z/Q_z = (1/R_1 + 1/R_3)/C_1(1+A) \quad (1)$$

where  $\omega_z^2$  is the constant term in the numerator,  $AK_1/R_1R_4C_1C_2(1+A)$ . The value of  $Q_z$  is therefore given by the expression

$$Q_z = [K_1C_1A(A+1)/R_1R_4C_2]^{1/2} R_1R_3/(R_1+R_3) \quad (2)$$

If the gain of the operational amplifier,  $A$ , is large (on the order of  $10^4$ ),  $Q_z$  is greater than 200. For such high values of  $A$ , the transfer function of the circuit in Fig. 2 is effectively

$$\frac{e_o(s)}{e_i(s)} = K_2 \frac{s^2 + [K_1/(R_1R_3C_1C_2)]}{s^2 + [s(R_3+R_5)/(C_2R_3R_6)] + K_2/(R_3R_4C_1C_2)} \quad (3)$$

Since this equation has no  $s$  term in the numerator, the transmission function has a deep notch at the frequency given by

$$\omega_z^2 = \frac{K_1}{R_1R_4C_1C_2} \quad (4)$$

The notch frequency may shift if component values drift, but the depth of the notch will not be materially affected by such drift.

### Calculating circuit performance

Equation (3) shows that the zero and pole frequencies, and  $Q_p$ , for the circuit in Fig. 2 are given by Eqs. (4) and

$$\omega_p^2 = \frac{K_2}{R_3R_4C_1C_2} \quad (5)$$

$$Q_p = \left[ \frac{K_2C_2}{R_3R_4C_1} \right]^{1/2} \frac{R_4R_5}{R_4+R_5} \quad (6)$$

$$\left[ \frac{\omega_z}{\omega_p} \right]^2 = \frac{K_1 R_3}{K_2 R_1} \quad (7)$$

The gain of the circuit in Fig. 2 at zero frequency is

$$\frac{e_o}{e_i} = \frac{K_1 R_3}{R_1} \quad (8)$$

and at infinite frequency is

$$\frac{e_o}{e_i} = K_2 \quad (9)$$

Thus,  $K_1$ ,  $K_2$  and  $R_3/R_1$  can be used to set the transfer gain below and above the zero frequency  $\omega_z$ .

Equations (7), (8), and (9) show how to select component values so that the circuit will function as a low-pass, high-pass, or notch filter; if  $K_1 = K_2 = 1$ ,

- $R_3$  greater than  $R_1$  gives a low-pass filter
- $R_3$  equal to  $R_1$  gives a notch filter
- $R_3$  less than  $R_1$  gives a high-pass filter

Figure 3 shows the result of interchanging  $R_1$  and  $R_3$  to convert the filter section from low-pass to high-pass.

Resistor  $R_5$  can be used to adjust  $Q_p$  without affecting the zero or pole frequencies; in fact, the circuit can be designed without  $R_5$ . If  $R_5$  is omitted,  $Q_p$  is

$$Q_p \Big|_{R_5 = \infty} = \left[ \frac{K_2 R_1 C_2}{R_3 C_1} \right]^{1/2} \quad (10)$$

Resistor  $R_5$  can then be added to lower the total  $Q_p$  if the application requires that the  $Q_p$  be adjustable; to get a given value of  $Q_p$ ,  $R_5$  should be

$$R_5 = \frac{1}{(1/Q_p)(K_2 C_2/R_3 R_1 C_1)^{1/2} - 1/R_1} \quad (11)$$

When  $R_5$  is included in the circuit its value should be of similar to that of  $R_1$ ; otherwise, the output of the op amp may saturate at the notch frequency. It is good practice to let  $R_5 = R_1$  when using  $R_5$  in the design. This value of  $R_5$  results in low sensitivity of  $Q_p$  to  $R_1$ , as shown below.

The source that drives one of these filter sections should have a low resistance. The source resistance has no effect on the notch frequency, but it does affect the overall gain,  $\omega_p$ , and  $Q_p$ .

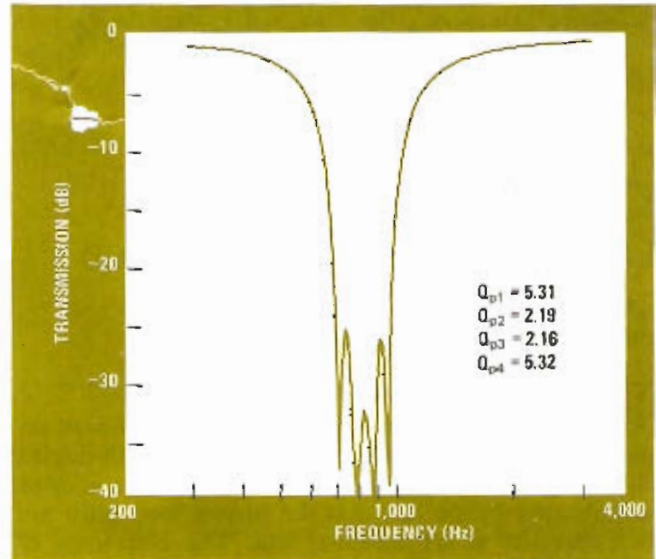
The equations defining the component values may be further simplified for notch filters, in which  $R_1 = R_3$  and therefore  $\omega_z = \omega_p = \omega_N$ , where  $\omega_N^2 = 1/R_1 R_1 C_1 C_2$  is the square of the notch frequency. Two sets of notch-filter equations that are useful to the designers are given in Table 1. One set of equations applies for the case where  $R_4$  is equal to  $R_5$ , and the other set applies when  $R_5$  is infinite;  $K_1 = K_2 = 1$  is assumed throughout.

Comparison of Eqs. 14 and 17 shows that the component value spread is reduced by a factor of four if  $R_5$  is set to infinity; but the flexibility in adjusting  $Q_p$  is lost.

To design a filter for a given  $\omega_N$ , an appropriate  $Q_p$  is chosen, and  $C_1$  is made equal to  $C_2$  at some fraction-of-a-microfarad value that yields convenient resistor sizes.

Figure 4 shows the response of a unity-gain notch section where  $f_N$  is 1,000 Hz and  $Q_p$  is varied over the range from 0.6 to 10 by varying  $R_5$ .

In applications where it is required to notch out a significant bandwidth, as in the band-rejection filter for a



5. Stopband. Four cascaded notch sections with different rejection frequencies and different  $Q_p$  values produce a bandstop filter.

TABLE 1  
NOTCH-FILTER EQUATIONS  
(FOR  $K_1 = K_2 = 1$ )

FOR $R_4 = R_5$		FOR $R_5 = \infty$	
$R_1 = R_3 = \frac{1}{2\omega_N Q_p C_1}$	(12)	$R_1 = R_3 = \frac{1}{\omega_N Q_p C_1}$	(15)
$R_4 = R_5 = \frac{2Q_p}{\omega_N C_2}$	(13)	$R_4 = \frac{Q_p}{\omega_N C_2}$	(16)
$\frac{C_2}{C_1} = 4 Q_p^2 \frac{R_1}{R_4}$	(14)	$\frac{C_2}{C_1} = Q_p^2 \frac{R_1}{R_4}$	(17)

TABLE 2  
COMPONENT SENSITIVITIES (FOR CIRCUIT OF FIG. 2)

	$R_1$	$R_3$	$R_4$	$R_5$	$C_1$	$C_2$	$K_1$	$K_2$
$\omega_z$	-1/2		-1/2		-1/2	-1/2	1/2	
$\omega_p$		-1/2	-1/2		-1/2	-1/2		1/2
$Q_p$		-1/2	$\frac{1 - R_4/R_5}{2(1 + R_5/R_4)}$	$\frac{1}{1 + R_5/R_4}$	-1/2	1/2		1/2
$Q_p$		-1/2	1/2		-1/2	1/2		1/2

Touch-Tone telephone receiver, individual notch sections can be cascaded. Figure 5 shows the response of four cascaded notch sections for such an application; the notch frequencies are 697, 770, 862, and 941 Hz. The frequencies from 700 to 1,000 Hz are rejected by 25 dB. And since resistor  $R_4$  is common to both  $\omega_z$  and  $\omega_p$ , the notch frequency is adjusted by trimming  $R_4$ .

The sensitivities of the singularities and of  $Q_p$  to fractional changes of passive-component values are shown in Table 2. For  $R_5 = R_4$  or for  $R_5$  equal infinity, all sensitivities are  $1/2$  or less, resulting in active filter sections that are stable with respect to component drift. □

FOR YEARS, PEOPLE WORKING ON CIRCUIT theory have been trying to do-in the inductor. This is especially true in the audio and sub-audio regions where inductors are inherently big, expensive, difficult to adjust, and subject to fields and hum pickup. After a lot of false trys and some rather poor ways of going about this, a batch of solid, reliable methods now exist that can do the job. Almost all these methods use low cost, readily available operational amplifiers. In most of the methods, the energy storage of an inductor is simulated by taking energy from a power supply and delivering it at the right point and in the right amount in a circuit to simulate exactly the behavior of an inductor.

Actually most methods don't work directly on replacing inductors. Instead, they look at the whole picture and attempt to come up with a *functionally equivalent* circuit that does exactly the same thing that the original one did, but internally does it in a wildly different way. These functionally equivalent circuits are often called *active filters*, and an active filter is simply any circuit that uses at least one operational amplifier or its equivalent to simulate exactly a circuit that normally would need at least one inductor to get the same result.

A filter itself is any frequency selective network. Three popular styles are the *low-pass* filter that passes only low frequencies and stops higher ones; the *bandpass* filter that passes only a few or a range of median frequencies; and a *high-pass* filter that allows only high frequencies to reach its output. A rumble filter on a turntable is a high-pass filter. The tuning on an AM radio is a bandpass filter, and the treble cut control on a hi-fi is a form of low-pass filter.

### A comparison

Before we go into the nuts and bolts details of how to build your own active filter, let's compare a simple active low-pass filter with an equivalent low-pass passive one (see Fig. 1). And, if we wanted to, we

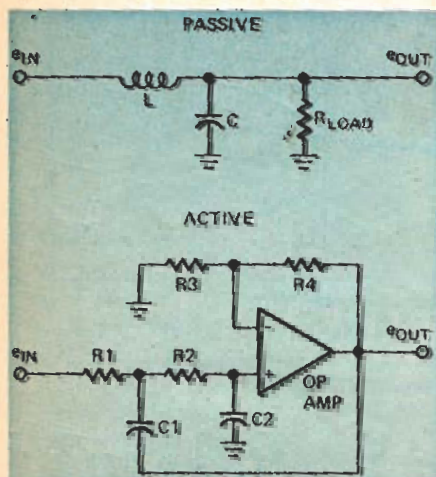


FIG. 1—LOW-PASS FILTERS. The passive L-C type is simpler, the active is more versatile.

could select the L-C ratio of the passive filter or the ratio of C1 and C2 in the active filter to get a response that looks like Fig. 2.

The way we get the response differs for the two circuits, but the result is the same. In the passive filter, the inductive reactance increases, and the capacitive reac-

tance decreases as we increase frequency, shunting more and more signal to ground. In the active filter, we essentially have two cascaded R-C sections at very high frequencies that also shunts the signal to ground. The problem is that if we left the amplifier out of the circuit, the response would droop very sloppily and very badly around the cutoff frequency.

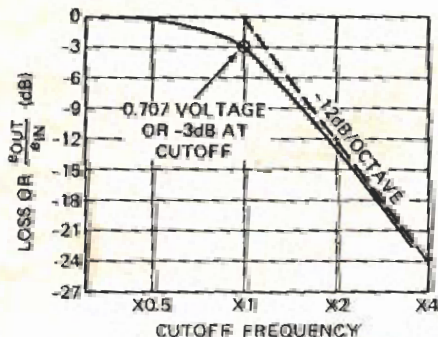


FIG. 2—RESPONSE OF LOW-PASS FILTER. Slope shown is 12 dB per octave.

What the op amp does is use circuit feedback via C1. It takes energy from the supply and introduces it in the middle of the R-C network to simulate exactly the same effect as energy storage in the inductor. Thus no R-C network by itself can ever hope to be as good as an L-C one, but an R-C network with some energy feedback controlled by an op amp is another story, and you can replace virtually any L-C network with a group of op amps, resistors, and capacitors. In fact, there's even things you can do actively that you can't with conventional circuits. Gain for instance.

We picked this particular response because it has the maximum possible flatness in the passband. It is called a *Butterworth* filter. If we try to steepen the response without adding any more parts, we'd get a hump in the passband, and the size of the hump would decide the *initial* but not the ultimate rate of falloff. Filters with humps are called *Chebyshev* filters, if the humps are in the passband and *Elliptical* filters, if the humps are both in the passband and the stopband. We could also make the response less flat and more gradual. This would improve the pulse response and overshoot at the expense of tilt in the passband and a more gradual rolloff. The

# How Active Filters Work

Here are full details on how to build filters with op-amps instead of inductors. This stable and reliable method works for practically all audio and sub-audio low-pass, bandpass, and high-pass designs

by DON LANCASTER

best of these is called a *Bessel* filter. The term that controls the shape of the filter near the cutoff frequency, but not at very low or very high frequencies is called the *damping* of the filter. The damping is controlled by the L/C ratio in the passive filter and the C1 to C2 ratio in the active filter, or by holding C1 and C2 constant and changing the ratio of R3 and R4.

We picked the Butterworth here because it is the most popular and the easiest to use. We'll stick with Butterworth filters all the way through this story. Other types are just as easy to build. All you have to do is move the damping and cutoff frequencies around a bit.

If we wanted something steeper than a 12-dB-per-octave rolloff, we'd have to add more parts. Two inductors and a capacitor would give you a 18-dB-per-octave filter, and two inductors and two capacitors would give you a 24-dB-per-octave rolloff, and so on. We call this the *order* of the filter. Second-, third-, and fourth-order filters have rolloff rates of 12, 18, and 24-dB-per-octave, and are the most popular normally used. Normally it takes one op amp for a second or third-order filter and two for a fourth.

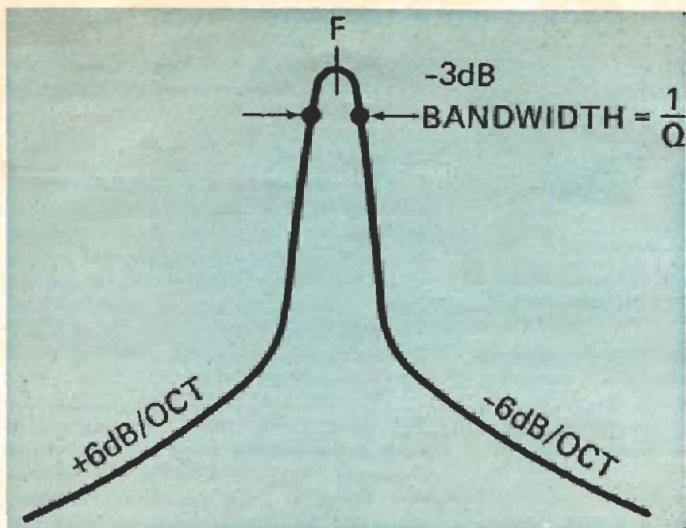
Note that the damping of the filter controls the response near cutoff, particularly the flatness in the passband, the time delay and overshoot, and the *initial* rate of falloff. The order of the filter controls the ultimate or *asymptotic* rate of falloff for the filter.

### Why go active?

The operational amplifier serves as a gain block with a very high input impedance and a very low output impedance. Its essential function is to provide for energy feedback to simulate the effect of energy storage in an inductor. Two nice benefits are the ability to drive any load and to use higher impedance (and almost always cheaper) components. So what are the benefits of an active filter? What do we gain and what do we lose when we go active?

The first and obvious thing we lose is the inductor, along with its cost, size, difficulty of adjustment, and sensitivity to hum and other magnetic fields. Note also that the passive filter has a load resistor. The value of this resistor is critical, for if you change it, the relative effects of the reactance changes of the inductor and capacitor





change and the response shape or the cut-off frequency may change. This is not true of the op amp active filter, for the op amp can drive most any reasonable load without changing the filter's response. We can vary the load from an open circuit down to anything the op-amp can reasonably drive without changing the response.

The input to the op amp is a very high impedance. This means you can use high-impedance resistors and capacitors for a given response at a given frequency. The benefits here are obvious. A high-impedance resistor costs the same as a low-ohms one, but a high-impedance capacitor is much smaller, and much cheaper.

Passive filters are inherently lossy, and the best we could expect to hope for would be slightly less than unity gain. With op amps and active filter designs, you sometimes can design for any circuit gain you want. Those we're going to show you have gains above unity.

Another big benefit is tuning. Large variable capacitors are nonexistent, while large variable inductors are expensive and a pain to adjust. On the active side, we have resistors R1 and R2 and surely changing them will change the response. For this particular circuit, we have to change both at once to change frequency without hurting the damping and response shape. This is easy to do with a dual pot, and we can easily get at least a 10:1 range. Even for slight tuning adjustments, the resistors are easy to change to get exactly the response you need. Because of this, active filters are generally more tuneable and easier to adjust than passive ones.

A final benefit is a bit subtle, but very important when we want a fancier higher order filter with faster cutoff slopes. We can cascade active filter blocks without any interaction, since they are free from fields and mutual inductance and since they generally have a high input impedance and a low output impedance. Cascadeability is a very big benefit. You normally can't simply cascade identical stages, for what was a -3-dB point becomes a -6 and so on. What you do is take the math expression for the higher order filter you want and factor it into second-order terms, and then build each second-order term separately. Generally, the individual block responses will be less damped and appear peaked when compared to the final result.

### Disadvantages and problems

If active filters are so good, why doesn't everybody use them? First and foremost, it's because very few people understand or appreciate what they are and what they can do. But, over and above this, there are some limitations and disadvantages to their use. Let's take a closer look.

Obviously, we need some supply power and the noise characteristics of the op amp can effect very low-level signals. More important is the high-frequency limitations of the operational amplifier. As you increase operating frequency, an op amp's open-loop gain decreases and its phase characteristics change so that you are limited to the upper frequency you can handle with a given operational amplifier.

For amplifiers like the 741 style or its dual and quad combinations, a reasonable upper frequency limit for active filters is between 20 and 50 kHz for low-pass and high-pass versions, and between 2 and 5

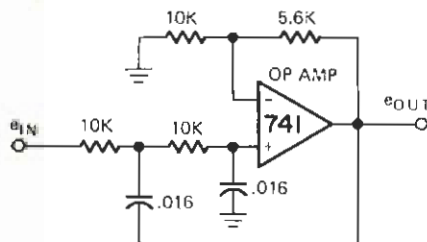


FIG. 3—ACTIVE FILTERS HAVE GAIN. Passive types are lossy circuits.

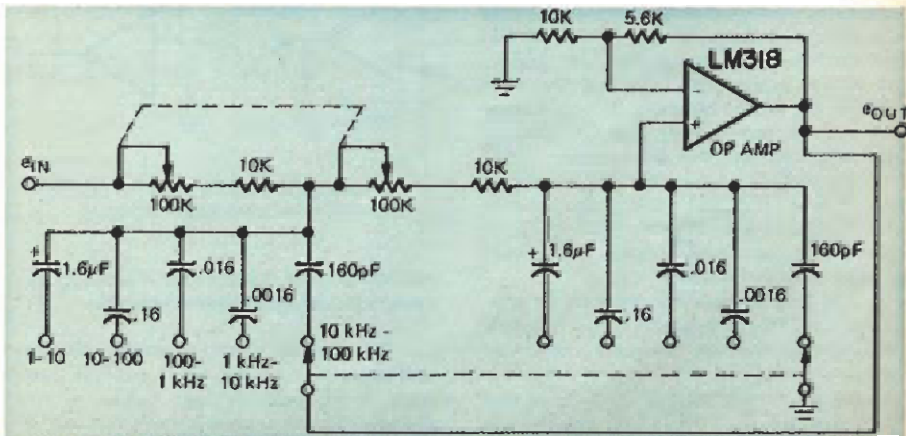


FIG. 4—AN ADJUSTABLE LOW-PASS FILTER covering from 1 Hz to 100 kHz in five frequency ranges. Switched capacitors select the bands, ganged potentiometers do the variable tuning.

kHz for bandpass designs. If you go to a higher performance internally compensated amplifier such as the National LM318, you can work up into the hundreds of kilohertz. Finally, if you go to really exotic op amps you can work higher, and even microwave active filter structures have been built. Thus, we have an upper audio limit for active filters built with the cheapest available op amps and a fractional megahertz limit for op amps in the \$3 to \$5 class.

Bandpass filters need more gain for resonance and thus are generally limited to lower frequencies. One way around the problem is to distribute the problem among two or more op amps so that each only has to provide some of the gain.

The low-frequency limit is another story. It's decided mostly by how much you want to pay for big capacitors and how high you're willing to let impedances get. With FET op amps, this can be a bunch, and operation down below 0.1 Hertz is certainly possible. Thus active filters are ideal for such sub audio work as brain wave research, seismology, geophysics, and fields like this.

One limitation, and the big one, is called the sensitivity problem. You have to ask how the individual components in the active filter are going to change the response if they are out of tolerance or drift with time. For instance, if a particular parameter such as a gain or a capacitance value happens to have a sensitivity of 0.5, the result is a 5% change in cutoff frequency or damping for a 10% change in component value. On the other hand, if a 1% variation makes a 50% change in something you've got problems. This is clearly ungood. When picking a way to build active filters, you have to be aware of the sensitivity problems and how to use them. The method we'll be showing you in a minute is very well behaved at fixed low gains and for lower to moderate Q bandpass designs.

A final limitation is one of method. There are about a dozen good and proven ways to design active filters. These all vary with their ease of understanding and what they can and cannot do. Some can't handle all three basic responses. Some allow single resistor tuning; others allow separate tuning of bandpass gain, center frequency, and Q. Some are well behaved at certain gains, but at others are too highly sensitive or actually unstable. You have to pick a method that works for you, is reliable, behaves well, and

does what you want it to. The one we'll be showing you is very easy to understand, stable and forgiving of component variations for fixed low gains, and useable in the bandpass case for low to moderate Q's. It usually takes two resistors simultaneously adjusted to tune, and in the bandpass case, you cannot separately set the Q, gain, and center frequency without a major change in components.

The method is called the *Sallen-Key* or Voltage Controlled Voltage Source (VCVS) method, and first appeared in the March 1955 *IRE Transactions on Circuit Theory*. Other popular filter methods are called the *Integrator Lag*, the *Biquadratic Section*, the *Multiple Feedback*, and the *State Variable*. Another type of active filter uses the *gyrator* or *impedance converter* but these generally take a bunch of parts and have a high-impedance output.

### Building your own

So, now we should know *why* we'd want to use active filters, and *where* to go to get complete design details, let's concentrate on *how* to actually build one. Here's a second-order Butterworth low-pass filter with a cutoff frequency of 1 kHz and a gain of 1.6 (see Fig. 3).

The response is identical to the curve in Fig. 2 with  $f=1$  kHz,  $2f=2$  kHz, and so on. As with any low-pass active filter, there must be a low dc impedance to ground at the source. Thus your source has to be less than 10,000 ohms and must provide a route to ground for the op amp's bias current. Again, the response is Butterworth, giving us the flattest possible passband, and an attenuation of -3 dB or 0.707 amplitude at the cutoff frequency, and smoothly falls off at -12 decibels per octave. This means that in the stopband as you double frequency, you get only one quarter the amplitude, and so on.

The above circuit looks deceptively simple and it is except, that a "magic" gain of 1.6 has been used that lets you use equal resistors, equal capacitors, and still have the desired shape. Change anything from the above, and the mathematics behave wildly. The circuit is forgiving of component variations and 5% components should be more than adequate for practically all uses.

To change frequency (in Fig. 1) you change R1 and R2 to identical values, or you simultaneously change C1 and C2 to new values. Raising R lowers the operating frequency. Raising C lowers the operating frequency. Thus, a 5000-ohm value instead of 10,000 ohms puts you at a 2 kHz cutoff frequency, and so on. A 0.032  $\mu$ F capacitor value puts you at 500 hertz and so on. If you change one capacitor, you *must* change the other. Similarly if you change one resistor, you *must* change the other, or the response shape will also change.

It's easy to see how we can use a dual pot to tune 10:1 and switch capacitors to get decade ranges. Fig. 4 shows a circuit that covers any cutoff frequency you want from 1 hertz to 100 kHz:

The pot rotation will generally be non-linear since the frequency varies inversely with pot rotation and resistance value. One way to linearize the pot is to use a dual audio log pot, with a normal taper if the dial is on the pot shaft and a reverse taper if the dial is on the panel.

If you just want one frequency differ-

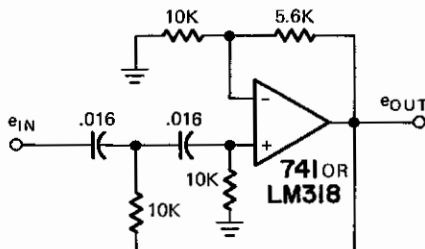


FIG. 5—IN HIGH-PASS FILTERS, the R and C shunt and series elements are transposed.

ent from 1 kHz, just calculate the capacitor value you need and change the capacitors, or change the resistors. It's simply the ratio of the capacitors equals the ratio of the frequencies and vice versa for the resistors.

### High-pass designs

The high-pass filter is a snap—you inside the circuit out and by a network principle called *duality* you're done. Fig. 5 is a 1-kHz Butterworth, second-order high-pass circuit.

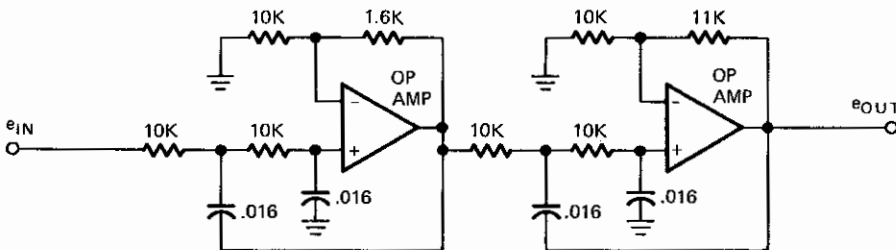
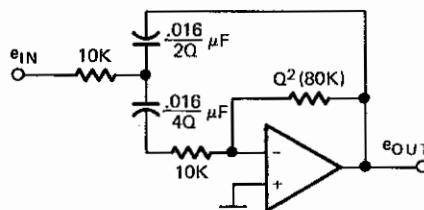
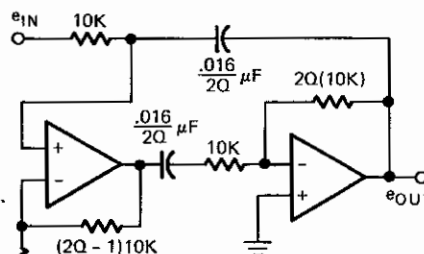


FIG. 6—TWO LOW-PASS SECTIONS IN CASCADE produce a fourth-order Butterworth filter with a rolloff slope of 24 dB per octave. The circuit's overall gain is about 8 dB.



OP AMP GAIN MUST GREATLY EXCEED  $8Q^2$  AT OPERATING FREQUENCY

CIRCUIT GAIN = 2Q



OP AMP GAIN MUST GREATLY EXCEED 2Q AT OPERATING FREQUENCY

CIRCUIT GAIN = 2Q

FIG. 7—ACTIVE BANDPASS CIRCUITS require high-gain operational amplifiers.

You simply interchange the resistors and capacitors on the input and you now have a highpass circuit. Again, if you change frequency, change both resistors or both capacitors to identical new values, or else the response shape will also change.

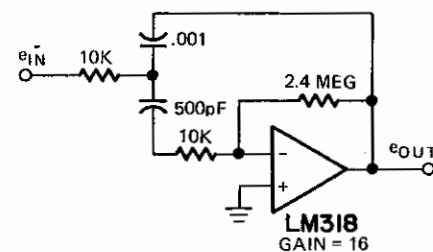
We can now see another big advantage to the "magic" gain value of 1.6—this circuit lets us switch from highpass to lowpass with a 4pdt switch without any change of component values.

### Steeper skirts

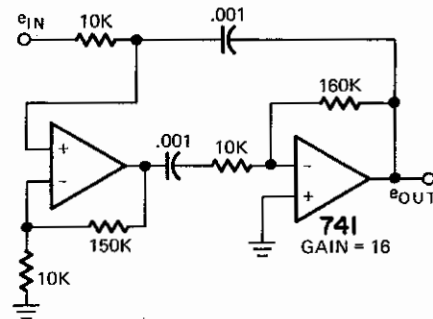
We can cascade two low-pass second-order sections to get a fourth-order Butterworth with a 24-dB-per-octave cutoff. We can't use identical sections, but we can make everything identical except for R4 (the feedback resistor) on each section. Finding the right R4 takes a lot of math, but here's the final circuit (see Fig. 6). It has an overall gain of 2.5:

The response is twice as good as before on a decibel scale. The passband is twice as flat and still drops only to -3 dB at the cutoff frequency of 1 kHz. The attenuation drops at 24-dB-per-octave, meaning that every doubling of frequency gives you only one-sixteenth the power and so on.

The higher performance circuit is somewhat harder to tune, since you simultaneously have to change four capacitors or



LM318  
GAIN = 16



741  
GAIN = 16

FIG. 8—CIRCUIT CONSTANTS for bandpass circuit where Q is 8 and frequency is 1 kHz.

four pots. Quadriphonic audio pots are a neat way to handle the tuning and they are reasonably available. Highpass to lowpass switching can be handled by a 8-pole-double-throw switch or two ganged 4-pole-double-throw pushbuttons, arranged so one is up when the other is down and vice versa.

Other orders and shapes of active filter  
(continued on page 71)

are just as easy to do.

**Bandpass designs**

Bandpass filters are generally much harder to design and more subtle to use. About all we have room for here is to show you two circuits that will do the job (see Fig. 7). They're shown for any Q at a center frequency of 1 kHz. And here in Fig. 8 are the same circuits for a Q=8: (1 kHz)

The two-amplifier job requires far less stable gain and works better for higher Q's and higher frequencies. Either circuit gives you the equivalent of a single series "pole" or tuned RLC circuit. This circuit, like its passive counterpart has a nasty feature that you must allow for. Its response starts falling off very steeply either side of resonance, but for very low or very high frequencies, it

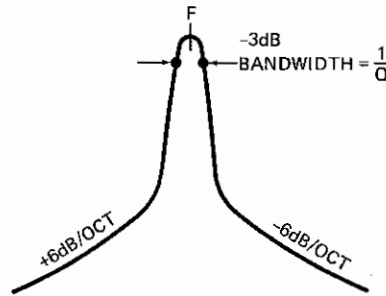


FIG. 9—BANDPASS RESPONSE CURVE measured at 3-dB point depends on circuit Q.

falls off at a more gradual rate of six decibels per octave. The response shape looks like the diagram in Fig. 9.

Normally, you cascade several poles to get the desired bandpass response. If we put the poles on top of one another, we get a very sharp response that is not very flat in the passband. We can control the response shape by staggering the poles in frequency and by altering their Q. Spreading the poles flattens out the passband, until finally you get a dip in the middle if you go too far. Another more formal way to design is to build a lowpass filter that does the job you want and then use a math process called *transformation* to get the desired bandpass shape. When you only need two poles, the simplest thing is to sit down with a breadboard and experiment with the Q and staggering for the response you need (the circuit moves around just like the low-pass and bandpass ones do by simultaneously changing capacitors or resistors); this is also a trivial problem for any computer that speaks BASIC, but the math is a bear otherwise. That's about all the details on bandpass design we have room for here. If enough readers are interested, we can put together another story with complete design curves for the two-pole bandpass designs in some other issue. **R-E**

**BUILD IN ACTIVE FILTER**

Next month in Radio-Electronics Don Lancaster presents complete details on how to build an active filter to meet your own needs. Don't miss it.

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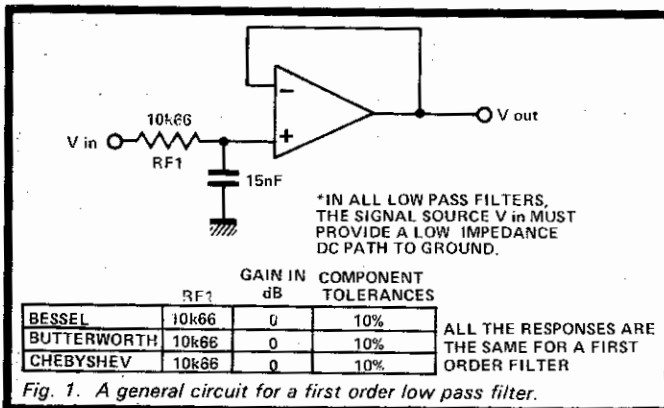
# DESIGNING & USING ACTIVE FILTERS PART 2

CONTINUING TIM ORR'S INSTRUCTIVE SERIES DESIGNED TO HELP THE HOME CONSTRUCTOR EMPLOY ONE OF THE MOST USEFUL CIRCUIT BLOCKS AVAILABLE

The following section contains all the information needed to be able to build low and high pass filters, of first, second, third and fourth order to Bessel, Butterworth and Chebyshev characteristics.

### Low pass

Figure 1 shows a first order low pass filter. In all the examples to follow the filters have been designed for 1kHz operation. Equal component value Sallen and



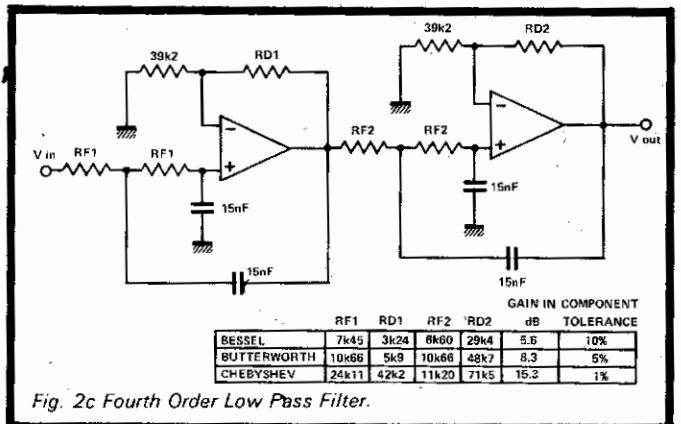
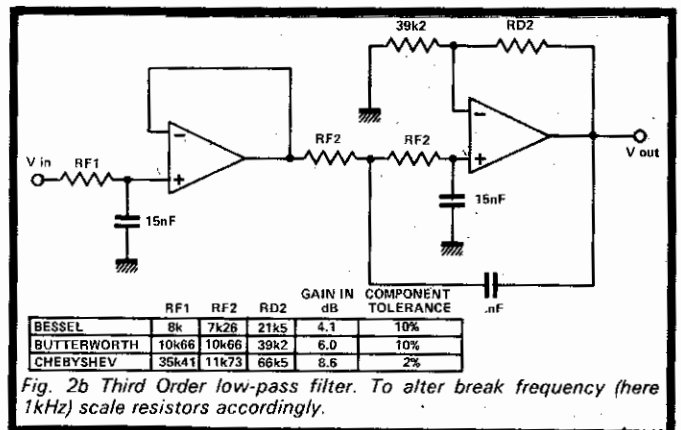
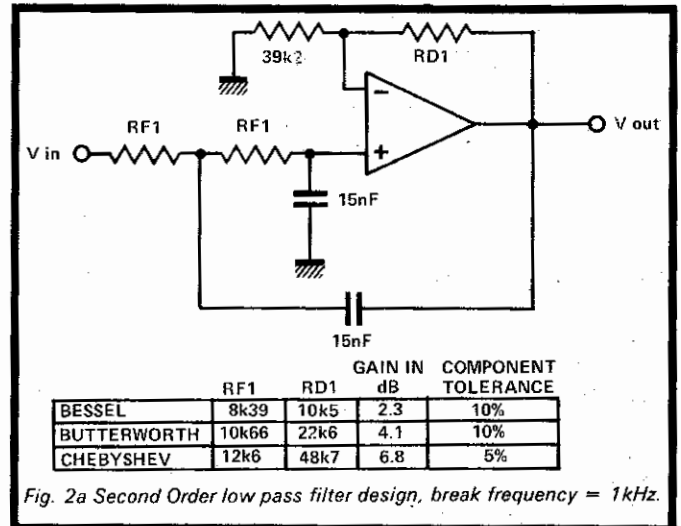
Key filters have been used as the basic building blocks. If operation at a frequency other than 1kHz is required, then the resistor/s R<sub>f</sub> should be scaled accordingly, (the R<sub>d</sub> resistors are not altered). For example, if operation is required at 250Hz, then the R<sub>f</sub> in the chart must be multiplied by

$$\frac{1000}{250}$$

which is  $\frac{\text{(Normalised 1kHz)}}{\text{Required frequency of operation}} = 4$ .

Figure 2 shows second, third and fourth order filters. The design procedure is as follows:—

1. Decide which type of filter is required, high, low, bandpass or notch.
2. In the case of high or low pass, decide which type of response is required, Bessel, Butterworth or Chebyshev.
3. Next, what filter order is needed. This will have led you to a particular order filter with components designed for 1kHz operation.
4. Scale the R<sub>f</sub> components so that the filter will operate at the required frequency.
5. Build and test the filter.



There are of course some problems which may occur. One is that these filters have a voltage gain in their passband. So you might find that although you have got the required frequency response there is an unexpected signal gain.

This may cause some problems with op-amp bandwidth. As a rule of thumb, the op amps should have 10 to 100 times more bandwidth than the product of the filters maximum operating frequency times the individual stage gain of each section. If the op amp runs out of bandwidth or introduces a phase shift then the filter is not going to work properly. For the examples given, if you use a 741 as the op amp then a frequency limit of approximately 10kHz should be imposed. (If an LM318 is used then the limit can go to 200kHz). Another problem is one of range of values of  $R_f$ . If  $R_f$  is made too small then large currents have to flow from the Op amp and this may effect the performance of the filter. If  $R_f$  is too large there may be hum pick-up problems and DC offset voltage problems due to bias currents. Therefore, keep  $R_f$  between 1k and 100k. If  $R_f$  needs to exceed this range, scale the capacitor as well.

### Charting examples

As an example of using the design tables, let us solve the following problem. Design an audio 'scratch' filter, having a break frequency of 7.5kHz and an attenuation at 15kHz of more than 20dB. The first decision to be made is what type of response do we want? A roll off of more than 20dB/octave is quite steep and so the Bessel filter is ruled out. The Chebyshev filter has a poor transient response and at 7.5kHz we would hear it ringing. Therefore a Butterworth response should be used. Next, the filter order. Third order gives us  $-18\text{dB/octave}$  which is not sufficient, fourth order gives  $-24\text{dB/octave}$ . Hence what is needed is a fourth order Butterworth design (fig. 2c).

The break frequency is 7.5kHz and so the resistors  $R_{f1}$  and  $R_{f2}$  have to be *divided* by 7.5. This gives  $R_{f1} = 1\text{k}42$ ,  $R_{f2} = 1\text{k}42$ ,  $R_{d1} = 5\text{k}9$ ,  $R_{d2} = 48\text{k}7$ ,  $C = 15\text{nF}$ , and the component tolerance is 5%. Now we must fit preferred values to the resistors.

$R_{d2}$  becomes 47k,  $R_{d1}$  becomes 6k2 (this is just over the limit of tolerance)  $R_{f1}$  and  $R_{f2}$  are a problem. Even when taken to the nearest E24 value they are outside the component tolerance allowed. There are two solutions; use the nearest E96 1% resistor or use 1k5. This will lower the break frequency by about 6%, but as this is only an audio filter no one will probably be any the wiser!

### High Pass

Figure 3 gives the design tables for high pass filters. The design procedure is exactly the same as that for low pass filters.

### Band Pass

Several second order band pass filters can be cascaded to produce a different response shape which, like those discussed earlier for the low and high pass filters, can be optimised to give maximum roll off, or maximum pass band 'flatness'. However, these tend to get rather difficult to design and so only second order filters will be discussed.

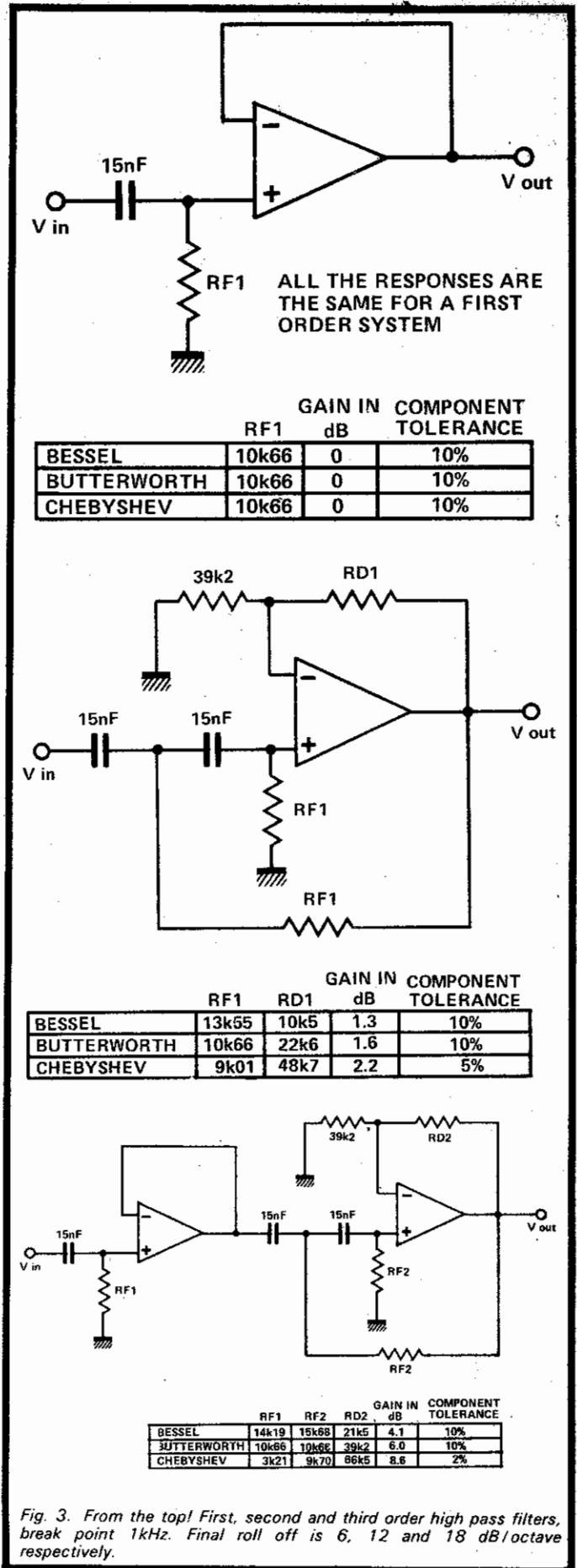


Fig. 3. From the top! First, second and third order high pass filters, break point 1kHz. Final roll off is 6, 12 and 18 dB/octave respectively.

# ACTIVE FILTERS

Figure 4 shows a simple bandpass filter known as a multiple feedback circuit. This circuit can only provide low values of Q up to about 5. It will probably oscillate if it is designed to give a higher Q. Note that a high Q implies a large gain at the centre frequency. Therefore care must be taken to ensure the op amp has enough bandwidth to cope with the situation. Fig. 4

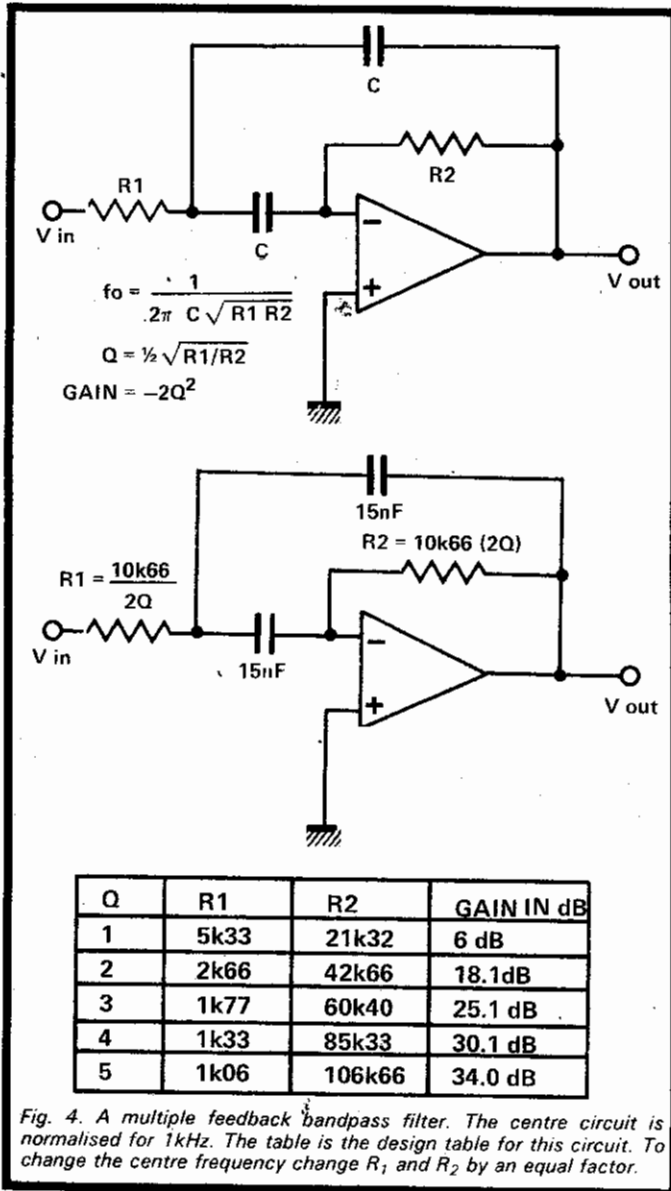


Fig. 4. A multiple feedback bandpass filter. The centre circuit is normalised for 1kHz. The table is the design table for this circuit. To change the centre frequency change  $R_1$  and  $R_2$  by an equal factor.

gives a design chart, normalised for 1kHz operation. First, choose a Q factor and then perform the frequency scaling. For instance, if the centre is 250Hz, then multiply both  $R_1$  and  $R_2$  by a factor of 4. If a high Q is required, then a multiple op amp circuit must be used. The 'state variable' and the 'Bi-Quad' are two such circuits and Q's as high as 500 may be obtained with them.

Figure 5 shows a state variable filter. It has three major features which are

1. It can provide a stable high Q performance.
2. It is easily tuned.
3. It is versatile, providing bandpass, lowpass and highpass outputs simultaneously.

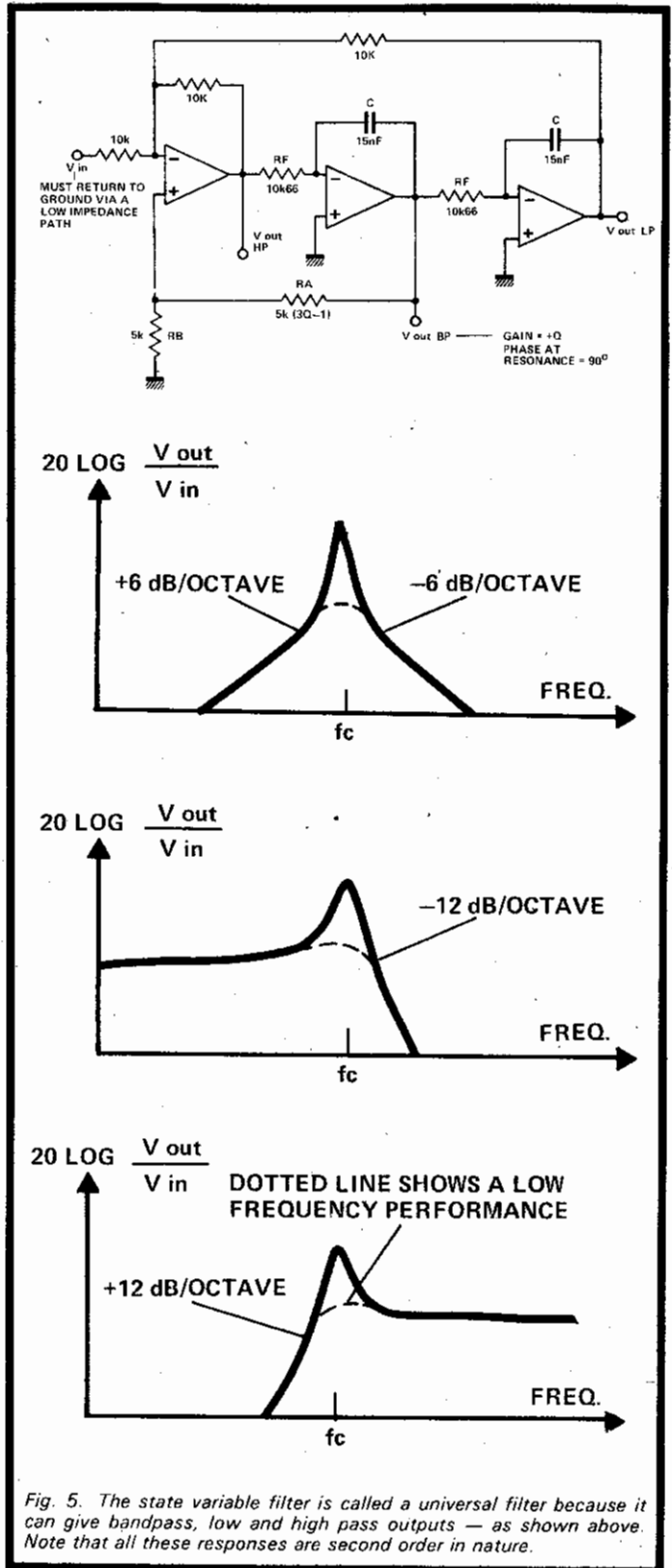


Fig. 5. The state variable filter is called a universal filter because it can give bandpass, low and high pass outputs — as shown above. Note that all these responses are second order in nature.

The Q is determined by the ratio of two resistors:  $R_A$  and  $R_B$ , where  $R_A/R_B = 3Q - 1$ . The resonant frequency  $f_c =$

$$\frac{1}{2\pi R_C C}$$

Note that there are two C's and two Rf's in the circuit, and so if the filter is to be tuneable, then both Rf's should change by an equal amount (the Rf's can be a stereo pot).

You will note that Q and fc are independent of each other, and so as the resonant frequency is changed, Q remains constant, and visa versa.

### Op amps

The requirements placed upon the op amps in the filter, Fig. 5, are less than that for the multiple feedback circuit. The op amps need only have an open loop gain of 3Q at the resonant frequency. Say we have a Q of 100 and an fc of 10kHz. Therefore the open loop gain is 300, the frequency is 10kHz and so the gain bandwidth product needed is 3MHz. When using a high Q, care must be taken with signal levels. The gain of the filter is +Q at resonance, and so if you are filtering a 1V signal with a Q of 100 then you could expect to get a 100V output signal!

National Semiconductors manufacture an active filter integrated circuit, which is a four amp network that can be used to realise state variable filters with Q's up to 500, and frequencies up to 10kHz. The device is called AF100.

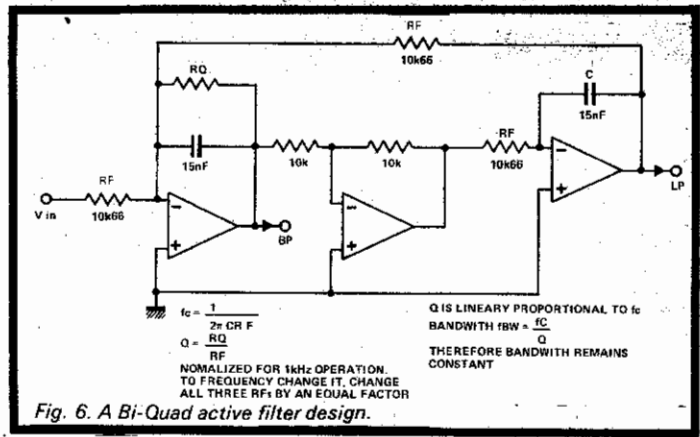


Fig. 6. A Bi-Quad active filter design.

Figure 6 shows a Bi-Quad active filter. It looks very similar to the state variable filter, but the small changes make it behave quite differently. It only has a bandpass and a low pass output. The resonant frequency is given by

$$f_c = \frac{1}{2\pi CR_1}$$

Next month: Comb filters, delay lines and some practical circuits to build up.

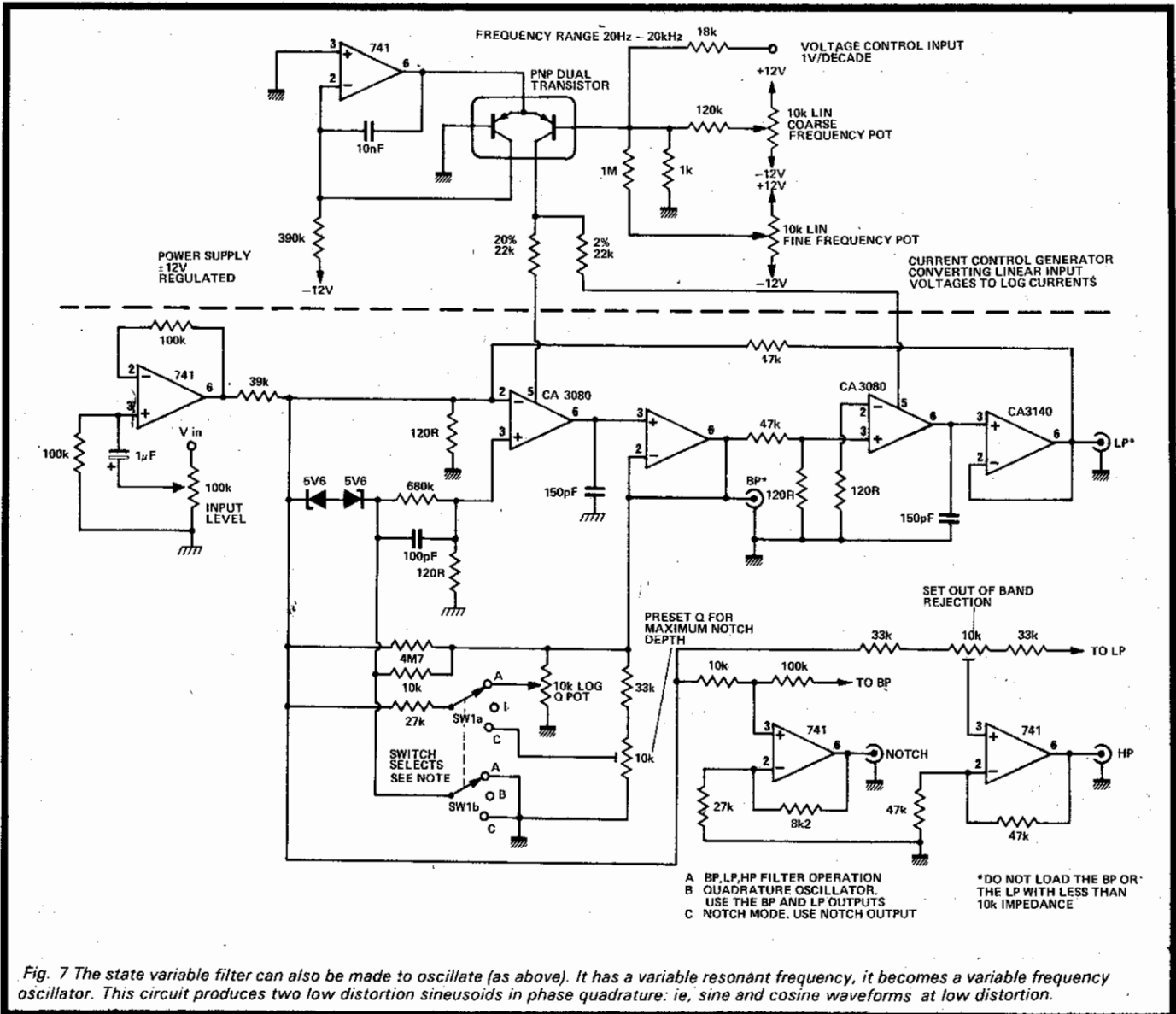


Fig. 7 The state variable filter can also be made to oscillate (as above). It has a variable resonant frequency, it becomes a variable frequency oscillator. This circuit produces two low distortion sinusoids in phase quadrature: ie, sine and cosine waveforms at low distortion.

# State-variable filter has high Q and gain

by Kamil Kraus  
Rokycany, Czechoslovakia

Although much work has been done in developing state-variable filters that use three or two operational amplifiers, earlier designs [*Electronics*, April 21, 1982, p. 126] suffer from a common disadvantage in that the filter's center frequency and the Q factor are interdependent. This new design provides a solution by introducing a parameter k in the transfer function of the filter.

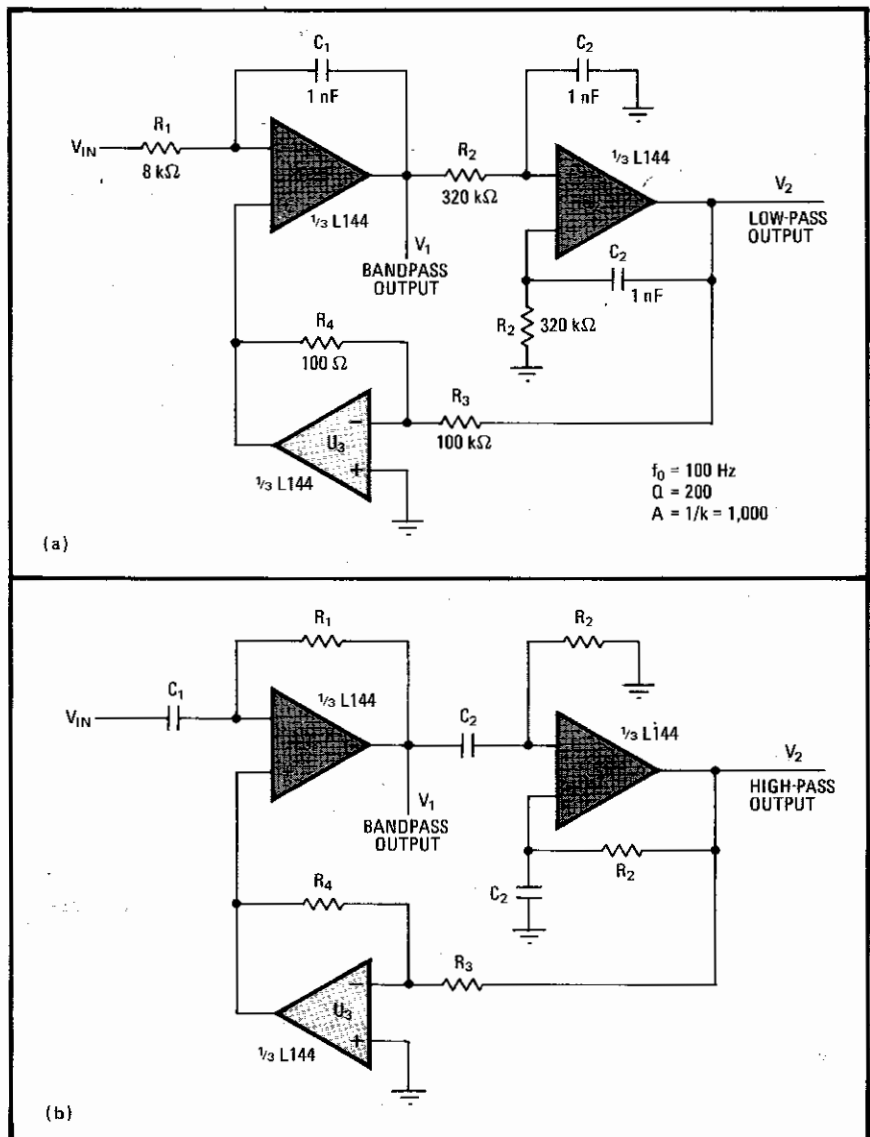
The circuit uses three op amps, U<sub>1</sub> through U<sub>3</sub>, to realize the filter. U<sub>1</sub> and U<sub>2</sub> function as negative and positive integrators, and U<sub>3</sub> completes the feedback loop. The second-order state-variable filter in (a) provides a bandpass output V<sub>1</sub> and a low-pass output V<sub>2</sub>. Its transfer function is  $V_1/V_{in} = -Qs/(1+s/Q+s^2)$  and  $V_2/V_{in} = -Qs^2/(1+s/Q+s^2)$ .

$= -1/k(1+s/Q+s^2)$  where  $s = j\omega/\omega_0$ ,  $1/\omega_0^2 = C_1C_2R_1R_2/k$ ,  $Q = (C_2R_2/kC_1R_1)^{1/2}$ , and  $k = R_4/R_3$ . The parameter  $\omega_0$  is the center frequency of the filter. Using  $C_1 = C_2 = C$  in the above equations,  $R_1 = 1/CQ\omega_0$  and  $R_2 = kQ/C\omega_0$ .

The filter in (b) gives a bandpass output V<sub>1</sub> and a high-pass output V<sub>2</sub>. As a result, its transfer function can be written as  $V_1/V_{in} = -Qs/(1+s/Q+s^2)$  and  $V_2/V_{in} = -(s/k)/(1+s/Q+s^2)$  where  $1/\omega_0^2 = kC_1C_2R_1R_2$  and  $Q = (C_1R_1/kC_2R_2)^{1/2}$ . Letting  $C_1 = C_2 = C$  yields  $R_1 = Q/\omega_0C$  and  $R_2 = 1/\omega_0kQC$ .

As an illustration, the filter in (a) is realized for a Q factor of 200 and a center frequency of 100 hertz. The design assumes a value of 1 nanofarad for C<sub>1</sub> and C<sub>2</sub>. With the equations shown above, the values obtained are R<sub>1</sub> = 8 kilohms, R<sub>2</sub> = 320 kΩ, R<sub>4</sub> = 100 Ω, and R<sub>3</sub> = 100 kΩ. The gain provided by the filter is A = 1/k = 1,000. The circuit uses components with 1% tolerances to reduce frequency drift and enhance circuit reliability. □

Designer's casebook is a regular feature in *Electronics*. We invite readers to submit original and unpublished circuit ideas and solutions to design problems. Explain briefly but thoroughly the circuit's operating principle and purpose. We'll pay \$75 for each item published.



**Independent.** With inverter U<sub>3</sub> in the feedback loop of the state-variable active filter, the filter's center frequency and Q factor can be tuned separately. The filter as shown in (a) gives bandpass and low-pass outputs, and the version in (b) gives bandpass and high-pass outputs.



# Active bandpass filter design is made easy with computer program

RCBAND time-shared software automatically determines the component values needed to build an active bandpass Butterworth or Chebyshev filter, in addition to predicting the circuit's frequency response

by Russell Kincaid and Frederick Shirley, *Sanders Associates Inc., Nashua, N.H.*

□ With the assistance of a computer and the right program, the often tedious task of designing filters can be considerably simplified. The time-shared program presented here, for instance, lets the computer crank out all of the numbers needed for an active band-pass filter design.

The program, which is called RCBAND, accurately computes the component values needed to provide the desired frequency response and permits the engineer to modify his initial design easily. RCBAND can produce either a Butterworth or Chebyshev filter characteristic, and its output data can provide both a tabulation and a graph of the predicted frequency response. The program is available\* without a surcharge from the User Program Library of Tymshare Inc., which is located in Palo Alto, Calif.

## Program structure

RCBAND employs the state-variable bandpass filter (see "Examining the state-variable filter," p. 125) as the basic filter section that is cascaded to produce the desired filter response. Up to 20 of these sections can be synthesized. The program normally analyzes the design as a series of single-pole sections, but custom filters can also be analyzed if the engineer specifies each section individually.

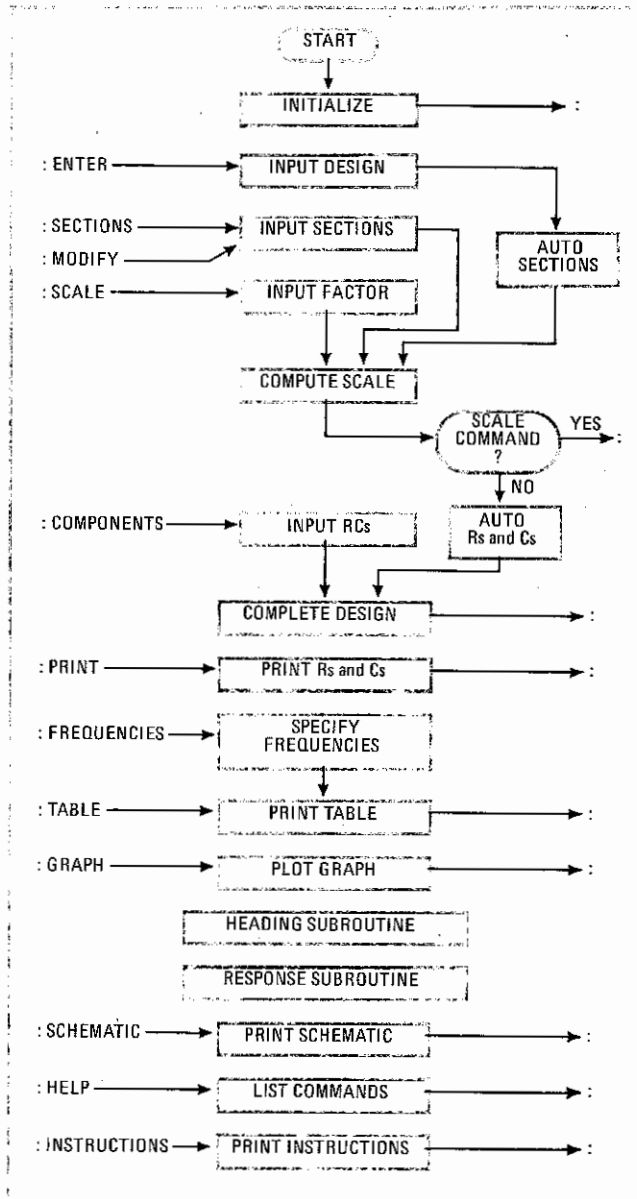
Only the basic filter specifications need be known to use the program—the number of poles desired, the type of filter characteristic, the center frequency, the bandwidth, and the gain. Or these inputs can be specified for each individual filter section, rather than for the entire filter.

The flow chart in Fig. 1 shows how RCBAND is structured. The program is written in NBASIC, the newest version of Tymshare's advanced SuperBASIC language. RCBAND is organized in a command-level format—that is, the computer prints a colon (: ) and waits for further

\*Copies of the RCBAND program listing can be obtained from Frederick Shirley, Sanders Associates Inc., 95 Canal St., M/S NCA1-6705, Nashua, N.H. 03060

## Closing the loop

Readers who wish to discuss the RCBAND computer program with the authors may call Russell Kincaid at (603) 885-4633 or Frederick Shirley at (603) 885-6239, during business hours the week of May 20.



**1. Filter design made easy.** The RCBAND computer program designs active bandpass filters by using the three-amplifier state-variable filter as a basic building block. As this flow-diagram shows, program control is returned to the user after each subroutine instruction is completed. RCBAND automatically computes component values, predicts frequency response, and permits easy modification.

## Designing bandpass filters

Bandpass filters can be made either by cascading low-pass and high-pass filter sections or by cascading stagger-tuned resonators. The low-pass/high-pass approach is best for wideband applications, while the stagger-tuned-resonator approach is best applied to narrowband filters.

The dividing line between the wide and narrowband approaches is determined by how much gain a section (a pole-pair) can provide and still have excess gain to operate the feedback loop. A two-resonator filter, for example, will have unity gain in the passband because the gain of one section offsets the skirt attenuation of the other. If the bandwidth of the over-all filter is increased by making the center frequency of one section higher or the center frequency of the other section lower, the filter gain will be decreased.

Another point to be considered in deciding between the two design approaches is the filter's skirt attenuation. For a resonator-type filter, the skirt attenuation is approximately 6 decibels per bandwidth octave, whereas for a low-pass/high-pass filter, the skirt attenuation is 6 dB per absolute octave. This means that a filter with a 2-kilohertz bandwidth centered at 10 kHz will provide an attenuation of 6 dB at 13 kHz if it's a one-resonator design. Or its attenuation will be 6 dB at 22 kHz if it's an equivalent low-pass/high-pass design.

The engineer must also choose between active and passive filter designs. Generally, an active filter is cheaper and lighter than a passive filter at low operating frequencies, specifically in the audio ranges, where an active filter works well. The frequency limitation of an active design lies in the amplifier.

The amplifier's performance can be estimated from its response curve of open-loop gain versus frequency. At those frequencies where amplifier gain is less than 60 dB, filter operation will be poor. And at higher frequencies, if amplifier gain is less than 40 dB, the filter will not work properly.

The low-pass pole existing in the amplifier tends to push the peak filter response to lower frequencies. Although this can be compensated for by trimming, the amplifier's pole frequency typically varies by 3:1 from unit to unit, so that trimming is difficult and rather temperature-sensitive.

Usually, the (Fairchild) 741-type operational amplifier

will work well up to about 1 kHz, and the (National Semiconductor) LM101-type op amp, which has feed-forward compensation, will operate satisfactorily at frequencies up to around 10 kHz. Even higher frequencies can be handled by dielectrically isolated op amps such as the (Harris) HA2600-type unit.

The filter-circuit configuration chosen to implement a single filter section will also have some effect on performance. A filter section can contain one, two, or three op amps. Although it's the most expensive to build, the three-amplifier approach has two distinct advantages over the other two circuits—it is the least sensitive to component variations and the easiest to tune. This three-amplifier circuit is commonly referred to as the state-variable filter.

At least three parameters are needed to specify the response of a bandpass filter: center frequency, bandwidth, and gain. These parameters are functions of resistance and capacitance, as well as amplifier gain and bandwidth. For optimum performance, it is best to make the parameters depend on resistor ratios and RC products only. This is because the temperature tracking of resistors and RC products is controllable, but variations in amplifier gain and bandwidth are not.

When the filter's design frequency approaches the amplifier's upper limit (the frequency where the gain is down to 60 dB), the sensitivity of the circuit to amplifier gain and bandwidth becomes increasingly important. High-Q designs require more amplifier gain and are less tolerant of frequency shifts caused by amplifier-bandwidth limitations.

Filter sections can be arranged in any order without affecting the over-all filter response. Also, the gain of one section may be raised, and the gain of another section reduced by a corresponding amount. The Q of a filter section tends to be highest at the band edges, and odd-ordered filters generally contain a low-Q center-frequency section.

These simple considerations can be used to advantage in particular applications. For example, in a three-stage low-level preamplifier filter, the center-tuned section would be first, providing high gain to raise the signal above circuit noise. The remaining sections could then have reduced gain to avoid the undesirable condition of overdrive distortion.

directions from the user after completing each subtask. This format is indicated in the flow diagram by the command-level symbol (a colon) at the input and at the output of each subroutine function.

Since all the input specifications are stored, the user has maximum freedom to initiate, modify, or investigate a design in a truly interactive time-sharing mode. A byproduct of this type of organization is a set of aid commands that the user can call for at any time if he needs assistance in working with the program. Two of these aid commands—HELP and INSTRUCTIONS—are shown in Fig. 2.

The HELP subroutine prints a list of all the commands available through the program's input command routine, and the INSTRUCTIONS subroutine prints a brief explanation of the RCBAND program. Another subroutine that is included for user convenience is the

SCHEMATIC listing, which is also shown in Fig. 2. It prints a circuit diagram of the standard state-variable filter section.

The front end of the program documents the run and initializes the variables, default actions, and user-defined number functions. Although initialization is not necessary in NBASIC, it does serve to increase computational efficiency if the program is to be compiled into a run-only machine-language version.

### RCBAND is easy to use

The default conditions override the normal NBASIC error messages in case of input error or a program bug. This enables RCBAND to maintain complete control of the interaction between the user and the computer so that the user need never be aware of the NBASIC source language while working with the program.

There are three user-defined number functions—they present computer-output data and accept user-input data in formats that are meaningful to the engineer. One function rounds numbers to the number of places specified by the user so that the printout contains only the number of digits required, regardless of number magnitude. A second function interprets user input in either scientific (1.23E3) or engineering (1.23 K) units. And the third function presents output data in engineering units that are five digits in length (101.86 K rather than 1.0186327E +05).

### Subroutines help the user

An input-command routine directs computation to one of several subroutines. At the completion of a subroutine, control is transferred back to the command level, the printer advances two lines, types a colon (:),

and waits for the user to type an alphanumeric command like TABLE or GRAPH.

The ENTER subroutine prompts the user and accepts the input specifications he enters at his computer terminal. An alternate method of entering input data is provided by the SECTIONS subroutine, which permits the user to define a custom filter by specifying each resonator individually. This subroutine is also used when changing a section by the MODIFY command. The AUTO SECTIONS subroutine computes the specifications for each state-variable filter section from the over-all design objectives given through the ENTER subroutine.

The frequency scale for the table and graph printouts is controlled by the SCALE subroutine. The last portion of this subroutine is called by both the ENTER and SECTIONS subroutines for the initial selection of the frequency scale. The COMPONENTS subroutine permits the

## Examining the state-variable filter

Although it requires three operational amplifiers, the state-variable active filter can operate at fairly high frequencies and can develop large values of Q. Moreover, the operating frequency, gain, and Q of this filter circuit are independent of each other, and the circuit itself is not overly sensitive to fluctuations in component values over the range of operation.

The state-variable filter consists of a summing amplifier ( $A_1$ ) and two integrators ( $A_2$  and  $A_3$ ). The circuit's bandpass response is taken at the output of integrator  $A_2$ . The natural frequencies,  $\omega_1$  and  $\omega_2$ , of integrators  $A_2$  and  $A_3$  are, respectively:

$$\omega_1 = 1/R_6C_1 \text{ and } \omega_2 = 1/R_7C_2$$

The transfer function of these integrators can be defined in terms of their natural frequencies (which are, of course, expressed in radians per second):

$$e_{out} = -\omega_1 e_1/s \text{ and } e_2 = -\omega_2 e_{out}/s$$

where  $s$  is complex frequency. Let three resistance ratios be defined for summer  $A_1$ :

$$\beta_1 = R_2/R_1 \text{ and } \beta_2 = R_2/R_3 \text{ and } \beta_3 = R_5/R_4$$

Then the transfer function of summer  $A_1$  can be written as:

$$e_1 = \frac{-\beta_1 e_{in} - \beta_2 e_2 + [R_4/(R_4 + R_5)]}{[1 + R_2(R_1 + R_3)/(R_1 R_3)]} e_{out}$$

Now the transfer function of the entire circuit can be expressed as:

$$e_{out}/e_{in} = \frac{\beta_1 \omega_1 s / [s^2 + [(1 + \beta_1 + \beta_2)/(1 + \beta_3)] \omega_1 s + \beta_2 \omega_1 \omega_2]}{(1 + \beta_3) \omega_1 s + \beta_2 \omega_1 \omega_2}$$

The principal filter specifications are center frequency ( $\omega_0$ ), damping factor ( $\zeta$ ), and gain constant ( $k$ ):

$$\begin{aligned} \omega_0 &= (\beta_2 \omega_1 \omega_2)^{1/2} \\ \zeta &= \frac{1}{2} [\omega_1 / (\beta_2 \omega_2)]^{1/2} [(1 + \beta_1 + \beta_2) / (1 + \beta_3)] \\ k &= \beta_1 \omega_1 \end{aligned}$$

These variables permit the filter's transfer function to be written in the standard format of a second-order equation:

$$e_{out}/e_{in} = ks / (s^2 + 2\zeta\omega_0 s + \omega_0^2)$$

The filter's 3-dB bandwidth ( $b_0$ ) and center-frequency

gain ( $g_0$ ) can be computed from this last equation:

$$\begin{aligned} b_0 &= 2\zeta\omega_0 = \omega_1 [(1 + \beta_1 + \beta_2) / (1 + \beta_3)] \\ g_0 &= \beta_1 [(1 + \beta_3) / (1 + \beta_1 + \beta_2)] \end{aligned}$$

A multiple-pole bandpass filter can be built as a series of one-pole-pair sections like the one shown here. Each pole pair in the over all filter transfer function is used to specify the resistor and capacitor values in the corresponding circuit section. The frequency response of any section is completely determined by center frequency  $\omega_0$ , 3-dB bandwidth  $b_0$ , and center-frequency gain  $g_0$ .

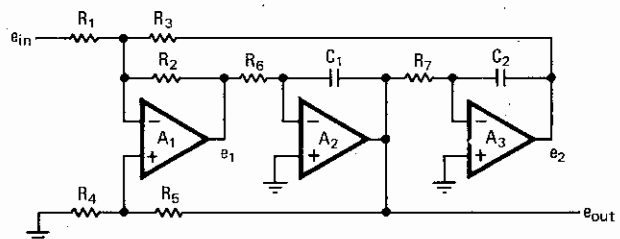
To simplify the computation, six of the nine unknown component values can be preset to reasonable values:

$$\begin{aligned} R_4 &= 1 \text{ kilohm} \\ R_2 = R_3 &= 10 \text{ kilohms} \\ C_1 = C_2 &= 0.01 \text{ microfarad} \\ R_6 = R_7 &= C_2 / C_1 \text{ ohms} \end{aligned}$$

These are the computer-selected component default values used by the RCBAND design program. The remaining three resistor values can be determined from these preset component values and the three known filter parameters:

$$\begin{aligned} R_1 &= [1/(\omega_0 C_2)](R_2/R_3)^{1/2} \\ R_1 &= [\omega_0 / (b_0 g_0)](R_2 R_3)^{1/2} \\ R_5 &= R_4 [(\omega_0 / b_0)(R_3/R_2)^{1/2} \\ &\quad [1 + (R_2/R_1) + (R_2/R_3)] - 1] \end{aligned}$$

The preselected values suggested for capacitors  $C_1$  and  $C_2$  are scaled by RCBAND to place the values of resistors  $R_6$  and  $R_7$  within the range of 10 kilohms to 100 kilohms. When necessary, the values of resistors  $R_2$  and  $R_3$  are also scaled, with resistor  $R_2$  decreasing in value and resistor  $R_3$  increasing in value so that resistor  $R_5$  is always positive in value.





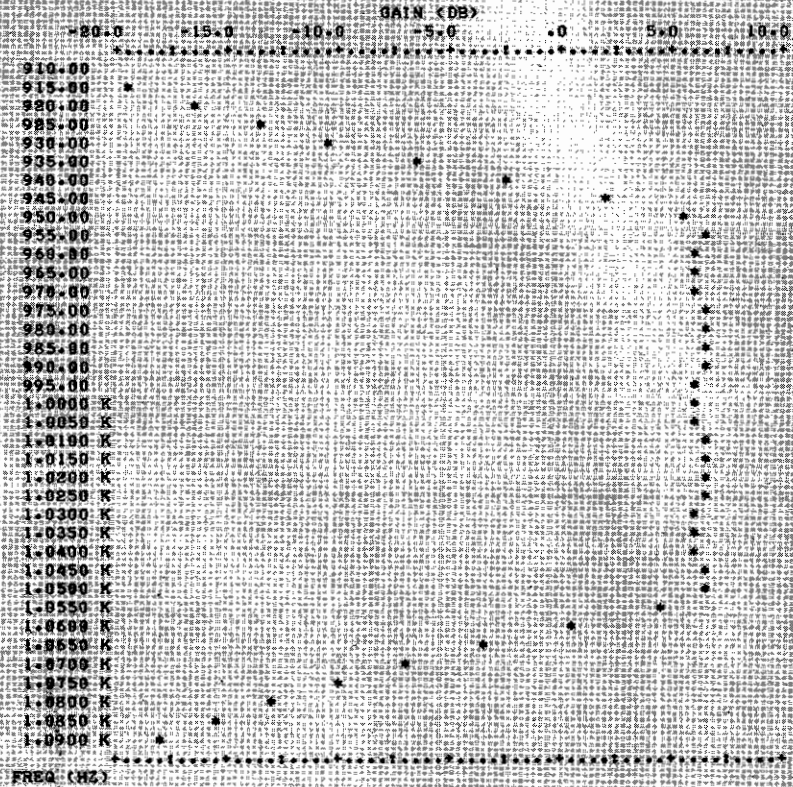
TABLE

CHEBYSHEV BANDPASS RC-ACTIVE FILTER  
 1.0000 KHZ CENTER      4 POLES  
 100.00 HZ WIDTH      6.02 DB GAIN  
 .5 DB RIPPLE

GRAPH  
 GAIN, PHASE OR TIME DELAY? GAIN  
 AUTOMATIC SCALING? NO  
 DB PER PRINT POSITION? .5  
 DB MAXIMUM? 10

CHEBYSHEV BANDPASS RC-ACTIVE FILTER  
 1.0000 KHZ CENTER      4 POLES  
 100.00 HZ WIDTH      6.02 DB GAIN  
 .5 DB RIPPLE

FREQ (HZ)	GAIN (DB)	PHASE (DEG)	DELAY (SEC)
910.00	-21.77	316.7	1757.3 U
915.00	-19.29	315.3	2103.7 U
920.00	-16.59	311.1	2563.7 U
925.00	-13.63	305.8	3084.9 U
930.00	-10.34	299.0	3681.5 U
935.00	-6.65	289.6	4250.8 U
940.00	-2.48	275.6	4737.5 U
945.00	1.98	252.7	5172.0 U
950.00	5.53	217.1	5583.9 U
955.00	6.52	178.6	5885.7 U
960.00	6.21	150.0	6147.0 U
965.00	6.00	128.3	6314.0 U
970.00	6.14	108.3	6374.0 U
975.00	6.38	89.3	6329.7 U
980.00	6.52	69.5	6194.0 U
985.00	6.46	50.2	6039.5 U
990.00	6.27	32.2	5868.6 U
995.00	6.09	15.7	5689.0 U
1.0000 K	6.02	0	5611.3 U
1.0050 K	6.09	-15.6	5799.2 U
1.0100 K	6.27	-31.9	5965.8 U
1.0150 K	6.45	-49.4	6098.0 U
1.0200 K	6.52	-64.0	6194.0 U
1.0250 K	6.40	-85.9	6250.8 U
1.0300 K	6.18	-105.4	6281.5 U
1.0350 K	6.03	-123.6	6281.5 U
1.0400 K	6.12	-142.9	6250.8 U
1.0450 K	6.44	-165.3	6194.0 U
1.0500 K	6.31	-197.9	6111.3 U
1.0550 K	6.27	-233.7	6039.5 U
1.0600 K	6.56	-261.4	6039.5 U
1.0650 K	-3.36	-279.0	7767.6 U
1.0700 K	-6.96	-290.5	8078.3 U
1.0750 K	-10.16	-298.6	8345.9 U
1.0800 K	-13.05	-304.7	8559.9 U
1.0850 K	-15.64	-309.5	8695.4 U
1.0900 K	-18.01	-313.3	8748.6 U



of 6.02 decibels, and a passband ripple of 0.5 db.

This data is given to RCBAND through the ENTER command. An unacceptable input—one that is ambiguous or out of range—is brought to the user's attention by an error routine so that corrected data can be reentered immediately. This prevents a computer run from aborting needlessly.

**Applying the program**

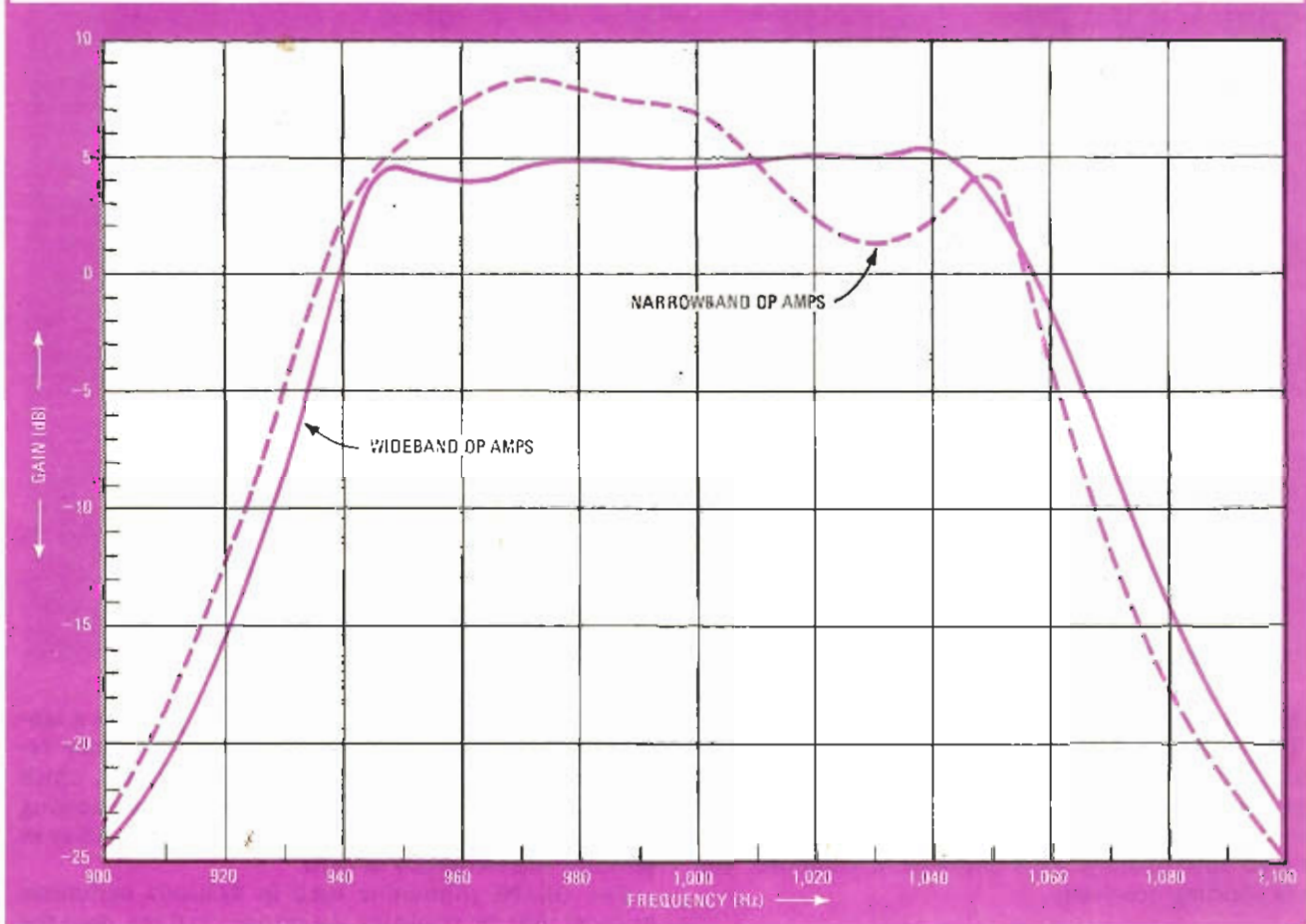
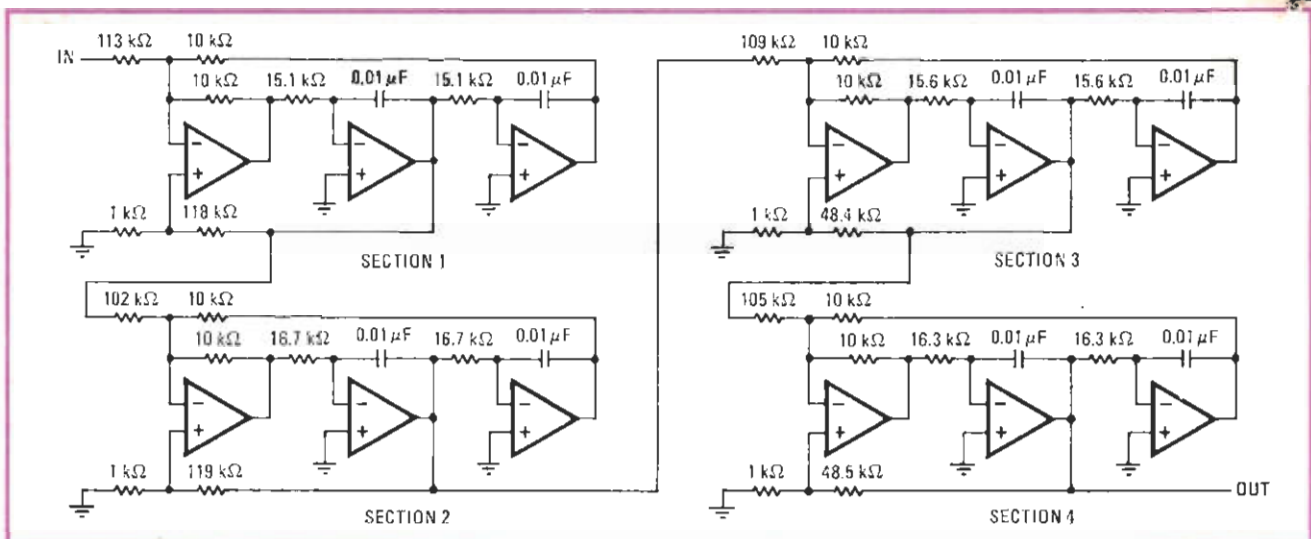
The PRINT command will provide a tabulation, as shown in Fig. 3, of the computed component values in each of the four filter sections. The section parameters (center frequency, bandwidth, and gain) are also printed to help the designer check and/or align the filter response. This printout, as well as the other major program printouts, are spaced on 8½-inch pages with a short heading on each page labeling the results. (The format is suitable for inclusion in engineering reports.) The data is presented in engineering units, rather than scientific notation, to make the printout easier for the engineer to interpret.

The TABLE and the GRAPH commands produce a tabulation and a plot, respectively, of the frequency response of the example filter. These two printouts, which are included in Fig. 3, give the response in engineering units—gain is in decibels, phase in degrees, time delay in seconds, and frequency in hertz.

The GRAPH subroutine used in RCBAND minimizes printout time by returning the printer carriage, once the actual data point is plotted, instead of wasting computer time by "printing" spaces all the way across the page. The user may select the scale for the gain axis, as done here, or he may let RCBAND determine it automatically. The scale for the frequency axis may also be user-selected through the SCALE command.

The schematic of the four-pole Chebyshev filter is drawn in Fig. 4, showing the final component values for all four filter sections. Needless to say, it still remains for the designer to select the operational amplifiers that are best for his application.

If narrowband 741-type operational amplifiers, which have a 60-dB bandwidth of 1 kHz, are used, the response



**4. The real thing.** Schematic shows the circuit configuration and component values of the four-pole Chebyshev filter described in the RC BAND printouts of Fig. 3. This filter's gain-versus-frequency response curve is closer to ideal when wideband op amps (like LM101-type units) are used than when narrowband op amps (like 741-type units) are used. Of course, the op amp chosen depends on the application.

curve drawn as a dashed color line in the gain-versus-frequency plot of Fig. 4 is obtained. On the other hand, if LM101-type operational amplifiers, which provide a 60-dB bandwidth of 10 kHz (with a 3-picofarad compensation capacitor), are used, the frequency response plotted by the solid color line results. Notice the similarity of the filter response predicted in the GRAPH printout of Fig. 3 to the filter response of the practical filter plotted in Fig. 4. □

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## State-variable filter uses only two op amps

by Charles Croskey

*Pennsylvania State University, University Park, Pa.*

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One of the more useful circuits for an active filter design—the state-variable active filter—can be somewhat expensive to build because it normally requires three operational amplifiers. Two of these op amps function as integrators, while the third is used as an inverter, since a difference integrator has been rather difficult to make with a normal op amp.

The state-variable filter in the diagram, however, re-

quires only two op amps. The circuit takes advantage of the recently introduced integrated quad amplifiers, such as Motorola's MC3401 and National's LM3900, which respond to a current difference instead of a voltage difference. Such amplifiers permit a difference integrator to be built simply.

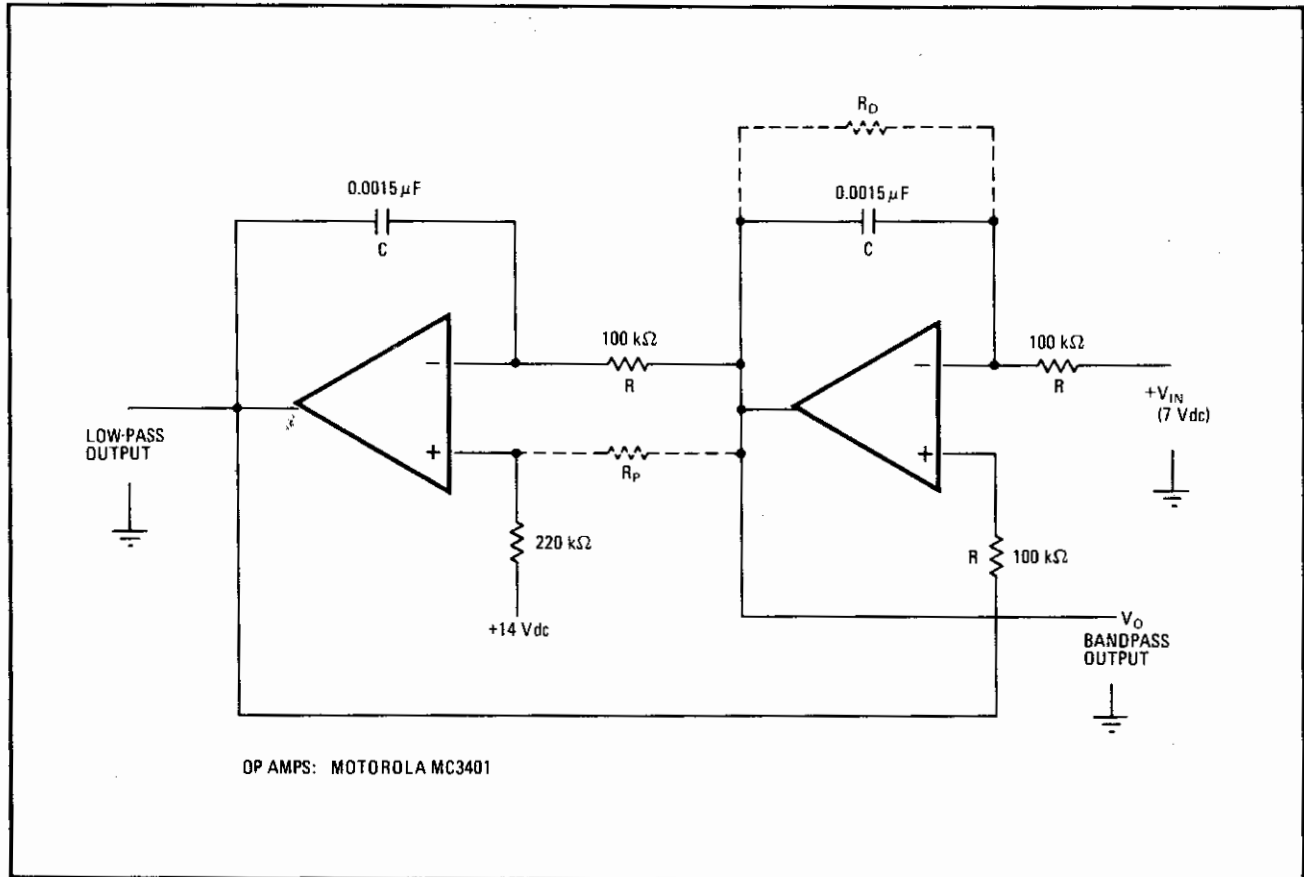
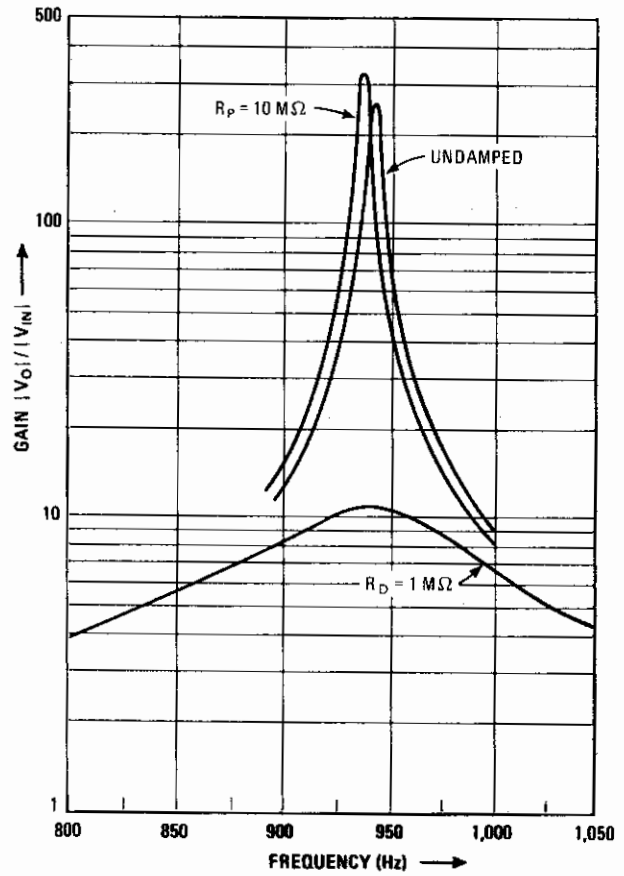
The center frequency of the filter's bandpass function is still determined by the usual relationship of:

$$\omega_0 = 1/RC$$

For the circuit values shown here, the center frequency is approximately 940 hertz. The filter's damping factor, and therefore its Q value, can be adjusted by resistors  $R_D$  and  $R_P$ . To increase the Q value, some positive feedback can be added through resistor  $R_P$ ; to decrease the Q value, resistive damping can be added by means of resistor  $R_D$ . As can be seen from the gain curves drawn in the figure, the Q value rises to 260 from a nominal (undamped) value of 248 when a 10-megohm resistor is used for  $R_P$ . Or if a 1-megohm resistor is used for  $R_D$ , the filter's Q value drops to 9.3.

Since the circuit requires only half of a quad amplifier package, the remaining two op amps can be employed as another filter or for additional gain. The filter also provides a low-pass output. □

**Eliminating an op amp.** This state-variable active filter employs only two op amps, instead of the three normally required. The usual inverter amplifier can be eliminated because the two op amps are connected as difference integrators. To adjust the filter's Q, resistor  $R_D$  or resistor  $R_P$  can be added to the circuit. The gain curves show both damped and undamped responses for the filter.





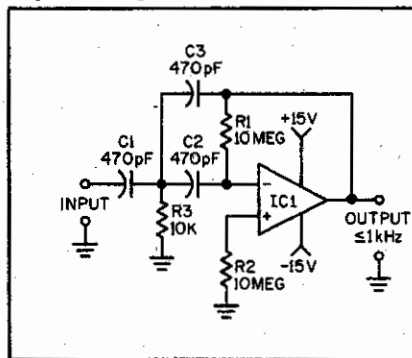
# IC 25

## Audio High Pass Filter

□ A high pass filter is a handy device to have around. Depending on the corner (turnover) frequency you select it can serve as a hum filter, distortion meter or highly-selective audio equalizer. The values of C1, C2, C3 and R1 provide a corner frequency of 1000 Hz. The IC has internal compensation so special wiring techniques are unnecessary. No pin connections are given because the 741 IC is available in many different pin configurations. Check the manufacturer's specs for the particular IC

used. R2 connects to the non-inverting (+) input of the IC, R1 between

the output and the inverting (-) input.



### PARTS LIST FOR AUDIO HIGH PASS FILTER

- C1, C2, C3**—470-pF, disc capacitor, 50 VDC or better
- IC1**—741-type operational amplifier
- R1, R2**—10 megohms, ½-watt resistor
- R3**—10,000-ohms, ½-watt resistor