Number 26 on your Feedback card

## Everyman's Guide to Active Filter Design

Now you know.

David Cripe KC3ZQ RR 2 Box 263 Camp Point IL 62320

ilters? Who needs filters? The answer is, everybody needs filters! Or at least every radio needs them! Electronic filters serve to pass signals within a desired band of frequencies, and reject signals lying outside this range. The most basic crystal radio contains at least one filter, and today's modern transceivers contain dozens. The ability to design filters for a given custom application is a valuable skill for the home-brew hobbyist. Good audio filtering on both the transmit and receive of your rig can really make a difference in the intelligibility of your signals. However, filter design is a topic not thoroughly covered in The ARRL Handbook, nor, for that matter, in many electrical engineering curricula. Designing any filter based on an existing circuit is a math-intensive process which is frightening enough, but to design a filter from scratch usually requires circuit optimization computer software which most hams will not have available. So what are we to do?

that standard tables of filter components would be extremely valuable for those engineers not wanting to derive these quantities for themselves. So, authors such as Blinchikov, Zverev, et al., have compiled wonderful books containing design information covering every conceivable filter you could imagine. Texts such as these contain tables of component values allowing the designer to construct filters consisting of inductors, capacitors, and resistors. The filter designer will discover, to his dismay, that the large-valued inductors required to construct a filter operating on signals in the audio frequency range are not stocked at the local Radio Shack<sup>®</sup>-nor anywhere else! This is where active filters are so valuable, as they allow the construction of filter networks composed of resistors, capacitors, op amps, and no inductors. Since it has not always been easy to come by information on how to convert a passive filter design, such as those contained in the filter design handbooks, to a more easily constructed active filter, I have had to figure much of this out for myself. I now have quite a bag of tricks allowing me to design an active filter, quickly, of nearly any topology I might select. Looking back at how much time this information could have saved me years

Luckily, the hard mathematical work involved in filter design has already been done for us. In the early days of digital computers, it was recognized **26** 73 Amateur Radio Today • March 1999 ago before I derived it, I am now sharing this so that others may benefit.

## Background

There is a certain vocabulary of terms used to describe filters, and it is useful to become familiar with them. A given filter has a *passband*, that range of frequencies it is designed to pass, and a *stopband*, that range of frequencies it is intended to attenuate. The *frequency response* of a filter is its ratio of output to input voltage versus frequency. The frequency at which a filter's output power is one-half that at the center of the passband is regarded as the point of transition between the passband and stopband, and is referred to the *-3 dB frequency*.

There are a number of different types of filters for different applications. The *low-pass* filter serves to pass signals below a given frequency, and block those above this. A *highpass* filter, conversely, blocks low-frequency signals, and passes highfrequency ones. A *band-pass* filter passes only those signals lying between



Fig. 1. Two-pole Sallen-Key low-pass filter.

two frequencies, while a *band-reject* filter serves to notch out those signals lying in a certain range.

Additionally, filters are further defined by the shape of their passband frequency response curves. A filter having the flattest possible frequency response within its passband is called a Butterworth filter. This is the most common type encountered in electronics design. A filter with the steepest possible transition between the passband and the stopband is called a Chebychev filter. However, this improvement comes at a price-the Chebychev filter exhibits ripple in its passband frequency response. The amplitude of the passband ripple of a given Chebychev filter, in dB, is used to describe it. A Bessel filter possesses a gradual roll-off of frequency response between passband and stopband. The Bessel filter frequency response is optimized for its time-domain response-which is to say that it does not "ring" in the manner of other filter shapes, such as the Butterworth and Chebychev. Consequently, the best CW filters are of the Bessel type.

low-pass or high-pass filter, the total number of reactive components (Ls or Cs) is the number of poles the filter contains. For a low-pass filter, the increase in attenuation for each octave of frequency increase in its stopband is 6 dB times the number of poles. In other words, a one-pole filter cuts the voltage of the signal passed in half each time its frequency is doubled. A threepole filter cuts the signal voltage to one-eighth with each doubling of frequency, etc. Consequently, the higher the number of poles a filter possesses, the higher the rate of attenuation within the stopband.

The filter design information contained in the filter handbooks is generally in a normalized format. These tables contain the values for capacitors and inductors for low-pass filters fed from a one ohm impedance source, terminated in one ohm at the output, and with a -3 dB point of  $1/2\pi$  Hz. This information is given for Bessel, Chebychev, and Butterworth filter shapes containing any given number of capacitors and inductors. From this normalized information, it is possible to derive component values for passive filters of any -3 dB frequency, whether



The amount of attenuation a filter provides within the stopband is a function of the number of inductors and capacitors it contains. For a simple

Continued on page 28

SHAPE	Rinput	X,	X <sub>2</sub>	R <sub>output</sub>						
Butterworth	0	1.414	0.707	1.0	-	Rinput	X1	-		$\rightarrow$
	1.0	1.414	1.414	1.0						Output Signal
0.1 dB Chebychev	0	1.404	0.829	1.0	R	Input Signal		± x2	Routput	
0.3 dB Chebychev	0	1.383	0.935	1.0	4				[	
1.0 dB Chebychev	0	1.301	1.195	1.0	$\downarrow$			$\checkmark$	$\checkmark$	
3.0 dB Chebychev	0	1.063	1.819	1.0						

Table 1. Normalized filter coefficients for two-pole filters.

73 Amateur Radio Today • March 1999 27



Fig. 2. Two-pole Sallen-Key high-pass filter:

## Everyman's Guide to Active Filter Design continued from page 27

of low-pass, high-pass, band-pass or band-stop types.

I have included Tables 1 through 4, which contain normalized low-pass filter component values for filters containing two through five total capacitors and inductors, in frequency responses having Bessel, Butterworth, 0.1, 0.3, 1.0, and 3.0 dB passband ripple Chebychev curves. (Notice that for the two- and four-pole filters, values are listed for an input impedance of zero ohms. This is because, for mathematical reasons, even-numbered-pole Chebychev filter responses cannot be attained with equally-terminated filters.)

its inventors. To convert from the normalized, low-pass values of **Table 1** to an actual working filter, we select the shape of the filter desired (Butterworth, Chebychev, or Bessel) using the singly-terminated values. After selecting the value of the -3 dB cutoff frequency,  $f_{.3dB}$ , the value of capacitor  $C_1$  is given by:

$$C_1 = \frac{X_1}{4\pi \cdot \mathbf{R} \cdot \mathbf{f}_{-3dB}}$$

and the value of C, is given by:

$$C_2 = \frac{X_2}{D_1 C_2}$$

$$\pi \cdot R \cdot f_{-3dB}$$

Let's try designing a 3 kHz,

values of 0.001  $\mu$ F and 0.0022  $\mu$ F for  $C_1$  and  $C_2$  respectively, we can work back to obtain R:

$$R = \frac{X_1}{4\pi \cdot C_1 \cdot f_{-3dB}} =$$

 $\frac{1.414}{12.6 \cdot 10^{-9} \text{ F} \cdot 3000 \text{ Hz}} = 37.4 \text{ k}\Omega$ 

This is not a standard resistor value, but we may use the next closest, 36  $k\Omega$ , which is less than 4% off.

That was easy enough! With these formulas and a calculator, anyone can design a low-pass filter. Now, how about a high-pass filter? Just as easy! For the Sallen-Key topology, a lowpass filter can be transformed into a high-pass filter simply by exchanging the resistors and capacitors. The same formulas hold true—just exchange the Rs and Cs! See Fig. 2:

$$R_1 = \frac{X_1}{4\pi \cdot C \cdot f_{-3dB}}$$
  
and  
$$X_2$$

## So much for theory! Are we ready to do some real designing?

The most common type of active filter is depicted in **Fig. 1**. It is a two-pole low-pass type, with two resistors, two capacitors, and an op amp. This very simple, easy-to-design circuit is referred to as a "Sallen-Key" filter, after Butterworth low-pass filter, such as might be used in an SSB receiver circuit. From **Table 1**, we see that the normalized value for  $X_1$  is 1.414, and the value for  $X_2$  is 0.707. With a little algebraic manipulation of the equations above, we obtain:

$$\frac{C_1}{C_2} = \frac{X_1}{4 \cdot X_2}$$

Thus, we see that for these values of  $X_1$  and  $X_2$ ,  $C_1 \div C_2 = 1/2$ . Approximating this ratio using common junk box

 $\mathbf{R}_2 = \frac{1}{\pi \cdot \mathbf{C} \cdot \mathbf{f}_{-3\,\mathrm{dB}}}$ 

Let's try designing a 300 Hz, twopole Butterworth high-pass filter. If we pick C = 0.01  $\mu$ F, then from the equations immediately above, R<sub>1</sub> = 37.4 k $\Omega$ , and R<sub>2</sub> = 75.0 k $\Omega$ .

The Sallen-Key active filter circuit is widely used for two-pole filter implementation. If, however, a filter with more than two poles is required, the design procedure for a Sallen-Key circuit implementation becomes much more difficult. Therefore, other circuit

SHAPE	R <sub>input</sub>	X,	X <sub>2</sub>	X <sub>3</sub>	R <sub>output</sub>				
essel	1.0	1.557	1.027	0.511	1.0				
utterworth	1.0	1.0	2.0	1.0	1.0	Rinput	×2	-	
1 dB Chebychev	1.0	1.435	1.594	1.435	1.0	-		1	
5 dB Chebychev	1.0	1.523	1.523	1.523	1.0	() input Signal =	X1	X	
dB Chebychev	1.0	1.686	1.400	1.686	1.0	V 5	-	4	
dB Chebychev	1.0	2.217	1.090	2.217	1.0				
) dB Chebychev	1.0	3.352	0.713	3.352	1.0				

Table 2. Normalized filter coefficients for three-pole filters.

28 73 Amateur Radio Today • March 1999



Fig. 3. Two-pole low-pass filter using Norton op amps in leapfrog topology.

topologies can be considered for these higher-order filters.

At this point, an introduction to a very useful op amp is appropriate. This is the Norton op amp, of which the type LM3900 is the most commonly used example. Whereas conventional op amps amplify the difference in voltage applied to their inputs, the Norton amplifier amplifies the difference in current applied to the input pins. The input pins of a Norton op amp have low impedance to ground, and so can be considered current sinks—the only caveat being that the input currents cannot be negative. Further, the LM3900 Norton op amp is designed to operate from a single supply voltage, so it is well suited for application in portable, 12-volt-operated equipment. Fig. 3 depicts the Norton op amp configured in a two-pole low-pass filter constructed in what is referred to as a leapfrog topology. Here, each capacitor or inductor in the passive filter prototype circuit is replaced by an op amp, a capacitor, and two (or three) resistors. The beauty of the leapfrog filter topology is that the formulas for calculating component values are very straightforward, allowing anyone with a hand calculator and filter tables to design a high-order active filter. While leapfrog filters can be constructed using conventional op amps, to implement a given filter it requires 50% more op amps and resistors than with Norton op amps. So for this discussion, we shall use Norton amplifiers.

We can now use this basic circuit to derive a series of equations to allow the design of an active filter. Selecting normalized filter values from Table 1, 2, 3, or 4 and cutoff frequency,  $f_{-3dB}$ , we then select a capacitor value for use Milder Conthroughout the filter. Unlike resistors, high-precision capacitors are not always easily available. However, capacitors of the same value, from the same manufacturing run, are usually matched in value unit-tounit better than the stated tolerance, and so can be used to obtain the necessary precision.



We refer to Fig. 3. The first op amp models the action of the first inductor in the passive low-pass circuit. The value of  $R_1$  is defined as:



www.ameritech.net/users/hamsure/ hamsure.html 800-988-7702 Available only in 48 contiguous US states Discount for AARA members.

CIRCLE 78 ON READER SERVICE CARD

SHAPE	Rinput	X,	X <sub>2</sub>	X <sub>3</sub>	X4	R <sub>output</sub>					
Bessel	0	1.540	1.114	0.855	0.400	1.0					
	1.0	1.736	1.629	0.780	0.613	1.0					
Butterworth	0	1.532	1.581	1.087	0.389	1.0	Rinput		×3	-	$\rightarrow$
	1.0	0.766	1.850	1.850	0.766	1.0	-				Output Signal
0.1 dB Chebychev	0	1.516	1.776	1.461	0.675	1.0	(U) Input Signi	al .	⊥ x2	+ X4	Routput
0.2 dB Chebychev	0	1.503	1.819	1.503	0.706	1.0	1				,
0.3 dB Chebychev	0	1.485	1.853	1.527	0.822	1.0	$\checkmark$		$\checkmark$		~
1.0 dB Chebychev	0	1.377	2.053	1.519	1.129	1.0					
3.0 dB Chebychev	0	1.102	2.635	1.281	1.793	1.0					

Table 3. Normalized filter coefficients for four-pole filters.

$$R_1 = \frac{X_1}{2\pi \cdot C \cdot f_{-3dB}}$$

If the filter is doubly terminated (i.e., having a resistor at the input), this first capacitor is paralleled by a resistor equal to R,

The second section of the filter mimics the action of the capacitor and resistor at the filter output. The value of resistor R, used in this section is given by:

$$R_2 = \frac{X_2}{2\pi \cdot C \cdot f_{-3dB}}$$

At this point check to see that these values are realistic-try to keep any values of R between 33 k $\Omega$  and 3.3  $M\Omega$  for best filter performance using the LM3900. If not, adjust the value of C accordingly, and try again.

This same technique can be extended to filters containing any number of poles. For a filter with more than two poles, we simply cascade successive sections, as is shown in

Subscriptions to 73 Magazine



Fig. 4. Five-pole low-pass filter using leapfrog topology.

Fig. 4. Let's try a more complicated example: a five-pole low-pass filter having a 0.21-dB-ripple Chebychev response, and a 3 kHz cut-off frequency. Let us also pick C to equal 0.0022 µF. From Table 4, we see that the five-pole, 0.21-dB-ripple Chebychev has the unique property that four of the five component values in the normalized filter are the same. This makes the design procedure very easy, as we only need to calculate two component values!

filter, both the first and last capacitor in the circuit are paralleled by a resistor of this value.

We calculate R, the same way:

$$R_3 = \frac{X_3}{2\pi \cdot C \cdot f_{-3dB}} =$$

2.393  $- = 57.4 \text{ k}\Omega$ . 6.28 · 2.2 · 10-9 · 3 kHz

Only \$24.97 - 1 year \$44.97 - 2 years \$65.00 - 3 years\*

\*best buy (54% off cover price!) Call 800-274-7373

> 73 Ad Sales Call 1-800-677-8838

Using our very first preceding equation to calculate R<sub>1</sub>, R<sub>2</sub>, R<sub>4</sub>, and R<sub>5</sub>:

$$R_1 = R_2 = R_4 = R_5 = \frac{X_1}{2\pi \cdot C \cdot f_{-.3dB}} =$$

$$\frac{1.475}{6.28 \cdot 2.2 \cdot 10^{-9} \cdot 3 \text{ kHz}} = 35.6 \text{ k}\Omega,$$

or the next closest standard value, 36 kQ. Recall that for this equally-terminated

or 56 kQ, the closest standard value.

This information should provide the average ham with the ability to design pretty much any filter he might need. This article is in no way an attempt to cover this topic fully-there are entire texts on the design of filters-and there is a limit to the quantity of material that can be presented in a magazine format. However, here's a starting point from which the novice filter designer may proceed on his own. Have fun-it's easy! 73

SHAPE	R <sub>input</sub>	X,	X <sub>2</sub>	X <sub>3</sub>	$X_4$	X <sub>5</sub>	R <sub>output</sub>					
Bessel	1.0	0.318	0.877	0.809	2.417	0.953	1.0					
Butterworth	1.0	0.631	1.604	2.032	1.604	0.631	1.0	Rinput	XZ	×4	n., .	$\rightarrow$
0.1 dB Chebychev	1.0	1.298	1.564	2.230	1.564	1.298	1.0	6				Output Signa
0.21 dB Chebychev	1.0	1.475	1.475	2.394	1.475	1.475	1.0		= ×1	± x3	= x5	Routput
0.3 dB Chebychev	1.0	1.600	1.406	2.495	1.406	1.600	1.0	4	$\checkmark$	$\triangleleft$	4 4	7
1.0 dB Chebychev	1.0	2.207	1.130	3.104	1.130	2.207	1.0					
3.0 dB Chebychev	1.0	3.483	0.763	4.540	0.763	3.483	1.0					

Table 4. Normalized filter coefficients for five-pole filters.

30 73 Amateur Radio Today • March 1999