

## by Ray Marston

Ray Marston takes an in-depth look at modern electronic filter circuits in this 3-part series.


high-pass filter.

C$-R$ and L-C filters are widely used in modern electronics. "They can be designed to either accept or reject specific frecjuencies or frequency bands and to ignore all others. Such filters can consist of nothing more than a number of capacitors and resistors (C-R types). or inductors and capacitors (l-C types). in which case, they are known as "passive" networks. Alternatively, they can consist of a C-R network combined with transistors or ICs to make what are known as active filters. This opening part of this 3 -part feature looks at the design of various $\mathrm{C}-\mathrm{R}$ passive filters; next month's episode will look at L-C filters, and the conclucling part of the series will present a selection of practical 'active' filter circuits. In all three articles, particular attention is given to the use of filters in instrumentation and test gear applications.

## Passive C-R Filters

Filter circuits are used to reject unwanted frequencies and pass only those wanted by the designer: In low-frequency applications (up to 100 kHz ), the filters are usually macle of C-R networks, and in high-frequency (aloove 100 kHz ) ones, they are usually made of L-C components. The two simplest C-R filters are the basic low-pass and the high-pass types.
A simple C-R low-pass filter (see Figure 1) passes low-frequency signals but rejects high-frequency ones. Its output falls by 3 dB (to $70.7 \%$ of the input value) at a break', 'crossover', or 'cutoff' frequency ( $f$ ) of $1 /(2 \pi R C)$, and then falls at a rate of $6 \mathrm{diB} /$ octave $(=20 \mathrm{~dB} /$ clecade $)$ as the frequency is increased. Thus, a 1 kHz filter gives about 12 dB of rejection to a 4 kHz signal, and $20 \mathrm{di3}$ to a 10 kHz one. The phase angle ( 0 ) of the output signal lags behind that of the input and equals -arctan ( $2 \pi R C$ ). or $-45^{\circ}$ at $f_{c}$.
A simple C-R high-pass filter (shown in Figure 2) passes high-frequency signals but rejects low-frequency ones. Its output is 3 dB down at a break frequency of $1 /(2 \pi R C)$, and falls at a 6 dB /octave rate as the frequency is decreased below this value. Thus, a 1 kHz filter gives 12 dl 3 of rejection to a 250 Hz signal, and 20 d 13 to 100 Hz one. The phase angle of the output signal leads that of the input and equals arctan $1 /(2 \pi \mathrm{fCR})$, or $+45^{\circ}$ at $f_{c}$.
Each of the above two filter circuits uses a single C-R stage and is known as a " 1 "-order filter. If a number ( $n$ ) of such filters are cascaded, the resulting circuit is known as an ' $n$ '"-order' filter and has a slope, beyond $\mathrm{f}_{c}$ of $(n \times 6 \mathrm{~dB})$ /octave. Thus, a 4 -order
1 kHz low-pass filter has a 24 di 3 /octave slope and gives 48 dB of rejection to a 4 kHz signal and 8 (clB to a 10 kHz one.

In practice, cascaded passive filters are rather difficult to design accurately, due to the distuptive interaction of neighbouring sections, and are rarely used in this simple form; instead, they are effectively cascaded by incorporating them in the feedlback networks of suitable op-amps, to make what are known as active filters. One instance where they are used, however; is as the basis of a so-called 'phase-shift' oscillator, as shown in basic form in Figure 3. Here, the filter is inserted between the output and

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Phose shift = 180}\mathrm{ at fo = = 1
VIN}/\mp@subsup{N}{\mathrm{ OUT }}{}=29\mathrm{ at fo
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Flgure 3. $3^{\text {rd }}$-order high-pass filter used as the basis of a phase-shift oscillator.

Figure 4.800 Hz phase-shift oscillator.


Figure 5. High-pass and low-pass filters cascaded to make a band-pass filter.


Figure 6. Balanced Wien tone filter.

input of the inverting ( $180^{\circ}$ phase shift) amplifier; the filter gives a total phase shift of $180^{\circ}$ at a frequency, fo, of about $1 /(14 \mathrm{RC})$, so the complete circuit has a loop shift of $360^{\circ}$ under this condition and will thus oscillate at this frequency if the amplifier has sufficient gain (about $\times 29$ ), to compensate for the losses of the filter and thus give a loop gain greater than unity.


Figure 4 shows a practical example of a 800 Hz version of the phase-shift oscillator: RV1 must be adjusted to give reasonable sine wave purity; the output level is variable via RV2.

## Band-pass and Notch Filters

A 'bancl-pass' filter is one that accepts a specific band or spread of frequencies but rejects or attenuates all others. A simple version of such a circuit can be made by cascacling a pair of $\mathrm{C}-\mathrm{R}$ high-pass and low-pass filters, as shown in Figure 5. The high-pass component values determine the lower break frequency: and the low-pass values set the upper break frequency.
Note that if the two filters in the above circuit are given the same break frequency value, the circuit becomes a tone-select filter, which gives minimum attenuation to a single frequency. Figure 6 shows a popular variation of this type of circuit, the Wien tone filter. R1 \& R2 and C1 \& C2 normally have equal values in this circuit, in which case, the circuit is said to be a 'balanced' type. The balanced Wien filter gives an attenuation factor of $3(=-9.5 \mathrm{~dB})$ at $f_{c}$ the circuit's major feature is that its output phase shift varies between $+90^{\circ}$ and $-90^{\circ}$, and is precisely $0^{\circ}$ at $f_{c}$. Consequently, the circuit can be used as the basis of a sine wave generator by simply connecting its output back to its input via a non-inverting amplifier with a gain of $\times 3$ (to give a loop gain of unity), as shown in basic form in Figure 7.
A 'notch' filter is one that gives total rejection of one specific frequency, but accepts all others. Such a filter can be made by witing the Wien network into the Bridge configuration shown in Figure 8. Here, R1 \& R2 act as a voltage divider with a nominal attenuation factor of 3 ; consequently, the voltage divider and the Wien filter outputs are identical at $f_{c}$, and the output (which equals the clifference between the two signals) is thus zero under this condition. In practice, the value of R1 or R2 must be carefully trimmed to give precise nulling at $f_{\text {c }}$.

The Wien Bridge network can be used as the basis of an oscillator by connecting it as in Figure 9 (a). At first glance, it might seem here that the Wien's output is fed to the input of a high-gain differential amplifier that has its output fed back to the Wien's input, to complete a positive feedback loop. When the circuit is redrawn as in Figure 9 (b), however, it becomes clear that the op-amp is actually used as a $\times 3$ non-inverting amplifier, and that this circuit is similar to that of Figure 7 In reality, these circuits must be fitted with some form of automatic gain control if they are used to generate good-quality sine waves.
A major feature of the Wien Bridge network is that its tuned frequency can easily be changed by simultaneously altering its two R or C values. Figure 10 shows this facility used to make a wide-range ( 15 Hz to 15 kHz ) variable notch filter, in which fine tuning and clecade switching are available via RV1 and SW1, and null trimming is available via RV2.


Figure 9. The Wien bridge oscillator in (a) is the same as that of (b).

## The 'Twin-T' Filter

Figure 11 shows another version of the notch circuit, the "twin-T" filter. Major advantages of this filter are that (unlike the Wien Bridge type) its input and output signals share a common 'ground' connection, and its off-frequency attenuation is less than that of the Wien. Its major disadvantage is that, if its tuning is to be made fully variable, the values of all three resistors (or capacitors) must be varied simultaneously. This filter is said to be a 'balanced' type when its components have the precise ratios shown; to obtain perfect nulling, the R/2 resistor value needs careful adjustment. Note in particular; that the circuit gives zero phase shift at $f_{c}$.

One weakness of the twin-T filter is that (like the Wien type) it has a very low effective ' $Q$ ' (quality) factor: $Q$ is defined as being the $f_{c}$ value divided by the bandwidth between the two -3 dB points on the filter's transmission curve, and in this case, equals 0.24 . What this means in practice, is that the filter sulbjects the second harmonic of $\mathrm{f}_{\mathrm{c}}$ to 9 dB of attenuation, whereas an ideal notch filter would give it zero attenuation. This weakness can casily be overcome by 'bootstrapping' the comion terminal of the filter, as shown in basic form in Figure 12. This technique enables high effective $Q$ values to be obtained, with negligible attenuation of the second harmonic of $\mathrm{E}_{\mathrm{c}}$.

The action of the balanced twin-T filter is fairly complex, as indicated by the operational representation of it, shown in Figure 13. It consists of a parallel-driven low-pass $\mathrm{f}_{d} 2$ and a high-pass $2 \mathrm{f}_{C}$ filter, with their outputs joined by an R-C ' $f_{c}$ ' voltage divider. This output divider loads the two filters and affects their phase shifts, the consequence being that the signals at points $A$ and $B$ have identical amplitudes but have


Figure 10. Variable frequency ( $\mathbf{1 5 \mathrm { Hz }}$ to $\mathbf{1 5 k H z}$ ) Wien Bridge notch filter.

phase shifts of $-45^{\circ}$ and $+45^{\circ}$ respectively at $f_{c}$; simultaneously, the impeclances of the $R$ and $C$ sections of the output divider are iclentical, and give a $45^{\circ}$ phase shift at $\mathrm{f}_{c}$. Consequently, the divider effectively cancels the two phase differences under this condition and gives an output of precisely zero, this being the phase-cancelled difference in amplitudes of the two signals.

Thus, a perfectly balanced twin-T filter gives zero output and zero phase shift at $f_{c}$. At frequencies just below $f_{c}$, the output is clominated by the actions of its low-pass filter, and is phase-shifted by $-90^{\circ}$; at frequencies just above $f_{c}$, the output is dominated by the actions of its high-pass filter, and is phase shifted by $+90^{\circ}$ (see Figure 11).

An 'unbalanced' version of the twin-T filter can be made by giving the ' R 2 ' resistor a value other than the ideal. If this resistor
has a value greater than $R / 2$, the circuit can be said to be positively unbalanced; such a circuit acts in a manner similar to that described, except that its notch has limited depth; it gives zero phase shift at $\mathrm{f}_{\text {c. }}$. If, on the orther hand, the resistor has a value less than $R / 2$, the circuit can be saicl to be negatively unbalanced; such a circuit also produces a notch of limited depth, but has the useful characteristic of generating a phase-inverted output, thus giving a $180^{\circ}$ phase shift at $f_{c}$, as shown in Figure 14.

Figure 15 shows how a negatively unbalanced twin-T notch filter can be used to make a $1 \mathrm{kH} \%$ oscillator or a tuned acceptance filter. The twin-T filter is simply wired between the input and output of the high-gain inverting amplifier, so that a loop shift of $360^{\circ}$ occurs at $f_{c}$. To make the circuit oscillate, RV1 is adjusted so that the twin-T notch gives just enough output to give the



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f_{c} \bumpeq \frac{1}{2 \pi R C}
$$

Figure 14. Negatively
unbalanced twin-T filter gives $180^{\circ}$ phase shlft at fC.
system a loop gain greater than unity; the circuit generates an excellent sine wave under this condition. To make the circuit act as a tone filter, RV1 is adjusted to give a loop gain less than unity, and the audio input signal is fed in via C 1 and R 1 ; under this condition, R1 and the twin-T filter interact to form a frequency-sensitive circuit that gives heavy negative feedback and low gain to all frequencies except $f_{c}$, to which it gives little negative feedback and high gain; the tuning sharpness is variable via RV1.

## C-R Component Selection

Single-stage C-R low-pass and high-pass filters and balanced Wien and twin-T networks all use the same formula to relate the $f_{c}$ value to that of $R$ and $C$, i.e., $f_{c}=1 /\left(2 \pi R_{C}\right)$ Figure 16 shows, for quick reference purposes, this formula transformed to enable the values of R or C to be determined when $f_{c}$ and one component value is known. When using these formulae, it is often easiest to work in terms of $\mathrm{kHz}, \mathrm{k} \Omega$, and $\mu \mathrm{F}$, as indicated.


Figure 15. 1 kHz oscillator/ acceptance-filter using negatively unbalanced twin-T network.

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\left.\begin{array}{r}
f_{C}=\frac{1}{2 \pi R C} \\
R=\frac{1}{2 \pi f_{c} C} \\
C=\frac{1}{2 \pi f_{c} R}
\end{array}\right\} \begin{aligned}
& f_{C}=k H z=k \Omega \\
& C=\mu F
\end{aligned}
$$

Figure 16. Formulae for finding the component values of single-stage high-pass or low-pass C-R filters and balanced Wien or twin-T networks.

